Exposure measures, scenarios and risk limits

Portfolio Management

1 A story: the dashboard in the car

- Exposure = value change per market move
- Read before the move, not after it
- Shares: one dial. Bonds: two. Options: three
- Dials let you gauge risk and then control it

IN THE LESSON

- Imagine driving a car with no dashboard.
- You would not know your speed, your fuel or your engine heat until something went wrong.
- A risk manager is in the same position without exposure measures.
- An exposure measure is a number that says how much a portfolio's value will change for a given move in a market factor, such as share prices, interest rates or volatility.
- Each type of asset has its own dials.
- Shares need one.
- Bonds need two.
- Options need three.
- The beauty of these dials is that they are read before the move, not after, so the manager can decide whether the car is going too fast for the road.

2 One dial, two dials, three dials

Exposure measures by asset

Shares: beta → Bonds: duration, convexity → Options: delta, gamma, vega

IN THE LESSON

- Here is the map for this section.
- Follow the three boxes from left to right.
- The first box is shares.
- A share position is described by its beta, which says how strongly it follows the market.
- The second box is bonds, which have duration and convexity.
- Duration is the straight line estimate of how a price reacts to a change in yield, and convexity corrects it for the curve.
- The third box is options.
- Delta is the first order reaction to the underlying price, gamma is how delta itself changes, and vega is the reaction to implied volatility.
- The more complicated the asset, the more dials we need, and each extra dial is one more term to add.

3 Example 1: a share position and its beta

Position 10m, beta 1.3, market falls 4%

Market return		**−4.0%**
Beta of the position		**1.3**
Expected position return	1.3 × −4.0%	**−5.2%**
Loss on 10m	10m × 5.2%	**520,000**

IN THE LESSON

- Example one.
- A fund manager at a made up firm called Harbour Capital holds ten million in shares with a beta of one point three against the market.
- A risk analyst asks what a four per cent market fall would do to the position.
- Beta is the exposure measure here.
- The expected return on the position is beta times the market return, which is one point three times minus four, so minus five point two per cent.
- On ten million that is a loss of five hundred twenty thousand.
- Notice how little work this took.
- One multiplication turned a market view into a money figure that can be compared with a limit.

4 Bond exposure: duration and convexity

$\%\Delta P \approx -\text{ModDur} \times \Delta y + \tfrac{1}{2} \times \text{Convexity} \times (\Delta y)^2$

Price change	Estimated price change
Duration term	Straight-line part: −duration × Δy
Convexity term	Curve correction: ½ × C × Δy²
Yield change	Change in yield, as a decimal

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- Now bonds.
- The percentage change in a bond's price is approximately minus modified duration times the change in yield, plus one half times convexity times the change in yield squared.
- The first term is the straight line.
- A bond loses about its modified duration, in per cent, for each one point rise in yield.
- The second term is a correction for curvature.
- Prices fall less than the straight line says when yields rise and rise more when yields fall, so the convexity term adds value in both directions.
- Two details cause most lost marks.
- The change in yield must be squared in the second term, and you must not drop the one half.

5 Example 2: a bond when yields rise

Mod. duration 7, convexity 60

Duration term, Δy = +1%	−7 × 0.01	**−7.0%**
Convexity term	½ × 60 × 0.01²	**+0.3%**
Estimate, Δy = +1%	−7.0% + 0.3%	**−6.7%**
Estimate, Δy = +3%	−21.0% + 2.7%	**−18.3%**
Trap: forgot the square	½ × 60 × 0.01	**+30%**

IN THE LESSON

- Example two.
- A bond portfolio has a modified duration of seven and a convexity of sixty.
- Yields rise by one percentage point.
- The duration term is minus seven times zero point zero one, which is minus seven per cent.
- The convexity term is one half times sixty times zero point zero one squared, which is zero point three per cent.
- So the estimate is minus six point seven per cent.
- Now suppose yields jump by three points.
- Duration says minus twenty-one, convexity adds two point seven, and the estimate is minus eighteen point three.
- Finally the trap.
- Forget to square the yield change at one point and the convexity term becomes thirty per cent, which is plainly absurd and a good warning sign.

6 Option exposure: delta, gamma and vega

$\Delta V \approx \delta \times \Delta S + \frac{1}{2} \times \Gamma \times (\Delta S)^2 + \text{vega} \times \Delta\sigma$

Value change	Estimated change in option value
Delta term	Delta × price change
Gamma term	½ × gamma × price change²
Vega term	Vega × volatility change

IN THE LESSON

- Options need three dials.
- Delta is the change in the option's value for a one unit move in the underlying price.
- Gamma is how much delta itself changes for a one unit move, so it is the curvature.
- Vega is the change in value for a one percentage point change in implied volatility, which is the market's expected price swing built into the option's price.
- Put them together and the change in the option's value is approximately delta times the price change, plus one half times gamma times the price change squared, plus vega times the change in volatility.
- Just as with bonds, gamma is to delta what convexity is to duration, so the same two slips apply: square the move and keep the half.

7 Example 3: a long call option

1,000 calls: delta 0.60, gamma 0.03, vega 0.25. Index +4, vol −2 points

Delta term	0.60 × 4	**+2.40**
Gamma term	½ × 0.03 × 4²	**+0.24**
Vega term	0.25 × −2	**−0.50**
Change per option	2.40 + 0.24 − 0.50	**+2.14**
Whole position	1,000 × 2.14	**+2,140**

IN THE LESSON

- Example three.
- The option desk at Harbour Capital holds one thousand call options.
- Each has a delta of zero point six, a gamma of zero point zero three and a vega of zero point two five per percentage point of volatility.
- Suppose the index rises by four points while implied volatility falls by two points on the same day.
- The delta term is zero point six times four, two point four.
- The gamma term is one half times zero point zero three times sixteen, zero point two four.
- The vega term is zero point two five times minus two, minus zero point five.
- Add them for two point one four per option, and across one thousand options the desk gains two thousand one hundred forty.

8 Example 4: hedged for delta, still exposed

Short 10,000 calls, delta-hedged. Share −6, vol +2 points

Book gamma / vega	−0.04 and −0.12 × 10,000	**−400 / −1,200**
Gamma loss	½ × −400 × 6²	**−7,200**
Vega loss	−1,200 × 2	**−2,400**
Total loss	delta term is zero	**−9,600**

IN THE LESSON

- Example four shows why one dial is never enough.
- A different desk is short ten thousand calls and has hedged delta exactly with shares, so first order price risk is zero.
- Each call has a gamma of minus zero point zero four from the desk's point of view, because it is short, and a vega of minus zero point one two.
- For the whole book, gamma is minus four hundred and vega is minus one thousand two hundred.
- Now the share falls six and implied volatility rises two points.
- The gamma loss is one half times minus four hundred times thirty-six, minus seven thousand two hundred.
- The vega loss is minus two thousand four hundred.
- The book loses nine thousand six hundred despite being delta hedged.

9 Sensitivities set limits, scenarios test events

- Bond desk: limit on portfolio duration
- Equity manager: limit on beta
- Options desk: limits on delta, gamma and vega
- Scenario measures: revalue under a stated event

IN THE LESSON

- Now how are these measures used in practice?
- Sensitivities are the language of everyday limits.
- A bond desk is told its portfolio duration must stay inside a band.
- An equity manager is told the fund's beta must not exceed a ceiling.
- An options desk has limits on delta, gamma and vega.
- They work on ordinary days because they are simple, additive across positions and quick to update.
- But a limit written in sensitivities says nothing about a crash where many things move at once.
- For that, firms use scenario measures, which revalue the portfolio under a specified set of market moves and report the resulting gain or loss.

10 Three ways to build a scenario

Historical or hypothetical

Pick the shocks → Revalue today's positions → Read the loss

Reverse stress test

Unbearable loss → Search for the moves → Judge how plausible

IN THE LESSON

- Scenarios come in three kinds, and the picture shows two routes.
- In the top row, historical and hypothetical scenarios run from shock to loss.
- A historical scenario takes the market moves of a past episode, such as a rate shock or an equity crash, and revalues today's positions as if it happened again.
- A hypothetical scenario designs the shocks instead, for an event that has never happened, or combines shocks on purpose, for example with correlations that rise in a crisis.
- The bottom row runs the other way.
- A reverse stress test starts with a loss the firm cannot bear, searches for the market moves that would cause it, and then asks how plausible those conditions are.

11 Example 5: a historical scenario on today's book

Today's book: 60m shares β 1.1, 30m govt dur 6, 10m corporate spread dur 5

Shares	60 × 1.1 × −25%	**−16.50m**
Government bonds	30 × 6 × 0.80%	**+1.44m**
Corporate bonds	10 × 5 × −2.00%	**−1.00m**
Scenario result	sum of the three	**−16.06m**

IN THE LESSON

- Example five.
- Hollins Capital, a made up manager, holds one hundred million: sixty million in shares with a beta of one point one, thirty million in government bonds with a duration of six, and ten million in corporate bonds with a spread duration of five.
- The risk officer replays a past crisis, in which shares fell twenty-five per cent, government yields fell eighty basis points and credit spreads widened by two hundred.
- Shares lose sixty times one point one times twenty-five per cent, sixteen point five million.
- Government bonds gain thirty million times six times zero point eight per cent, one point four four million.
- Corporates lose ten million times five times two per cent, one million.
- The total is a loss of sixteen point zero six million.

12 Example 6: working backwards

Unbearable loss 12m. Which share fall causes it?

Unbearable loss	set by the board	**12m**
Effective share exposure	60 × 1.1	**66m**
Share fall that does it	12 ÷ 66	**18.2%**
Past crisis for comparison	already happened	**−25%**

IN THE LESSON

- Example six uses the same book for a reverse stress test.
- Suppose the board says that a loss of twelve million is the most the firm can bear.
- Reverse stress testing asks what would cause that.
- Take shares alone.
- The effective exposure is sixty million times beta one point one, sixty-six million.
- To lose twelve million, shares would need to fall twelve divided by sixty-six, which is eighteen point two per cent.
- The next question is the useful one.
- Is an eighteen per cent fall plausible?
- Yes, it is smaller than the crisis we just replayed.
- So the firm learns that its pain threshold sits inside moves it has already seen, and it can decide to cut the exposure or hedge.

13 The trap: yesterday's portfolio

- A historical scenario is moves, not positions
- Apply the old moves to the book held today
- New instruments need a mapped shock
- Old positions only tell you about the past

IN THE LESSON

- There is one trap here that catches many people.
- A historical scenario is a set of market moves, not a set of positions.
- If a firm applies the old episode to the portfolio it held back then, it learns how it would have fared, which is history.
- The risk question is how today's book would fare.
- Today's portfolio may hold different instruments, larger positions, or securities that did not exist during that episode, such as new green bonds.
- So always ask the same question.
- Take the shocks from the past, but revalue the positions held today, and make sure every new instrument has some way of being mapped to a shock.

14 Duration is exact near zero, wrong far out

Chart: Price change (%) vs Change in yield, −3 to +3 points

- Duration 10, convexity 150
- Duration only: −30%
- With convexity: −23.3%
- Same near zero

IN THE LESSON

- Here is the weakness of a sensitivity as a picture.
- The horizontal axis is the change in yield, from minus three to plus three percentage points.
- The vertical axis is the price change of a bond with a duration of ten and a convexity of one hundred fifty, in per cent.
- The straight line is the duration estimate alone.
- The curve adds convexity.
- Near zero the two are almost the same, which is why duration is so good for limits and everyday hedges.
- Move out and they part.
- For a three point rise the straight line says minus thirty, while the curve says about minus twenty-three.
- For a fall, duration understates the gain.

15 Sensitivity versus scenario: the main flaw

Each measure fails in a predictable direction

Measure	Main flaw
Sensitivities	Local
Historical scenario	Backward
Hypothetical scenario	No odds
Reverse stress test	Judgement

IN THE LESSON

- Now the full picture, one measure at a time.
- Sensitivities are simple, precise for small moves and easy to hedge against.
- But they are local.
- Duration and delta ignore curvature for large moves, each one covers a single factor, and none attaches a probability.
- Historical scenarios are objective and easy to explain to a board, but the next crisis is unlikely to repeat the last, today's instruments may not have existed then, and the old correlations may not return.
- Hypothetical scenarios can explore new risks and breakdowns, but they depend on judgement, are hard to calibrate and carry no probability.
- Reverse stress tests expose the weak points, but also rest on judgement.

16 Strengths worth keeping

- Sensitivities: precise, additive, easy to hedge
- Historical: objective, easy to explain
- Hypothetical: can explore new risks and correlation breaks
- Best used together, with a probability-based measure

IN THE LESSON

- None of this means the measures are weak.
- It means each does a different job.
- Sensitivities are the tool for daily control: precise, additive and easy to hedge, so a desk can offset an exposure the moment it appears.
- Historical scenarios are credible because they actually happened, and a board understands them instantly.
- Hypothetical scenarios stretch the imagination, so they can capture a new kind of event or a correlation breakdown, where assets that normally diversify fall together.
- Reverse stress tests find the weak spot nobody thought to test.
- A sensible risk framework therefore combines them, and also uses a statistical measure such as value at risk, which does attach a probability to a loss.

17 Mistakes to avoid so far

- 1. Convexity, gamma: square the move, keep the half
- 2. Sensitivities are local: big moves need the curve
- 3. Use today's positions in a historical scenario
- 4. A delta-hedged book still has gamma and vega
- 5. Surviving some scenarios is not being safe

IN THE LESSON

- Let us pause on the common mistakes.
- First, in the convexity or gamma term, forgetting to square the change or dropping the one half.
- Second, treating a sensitivity as if it were true for a big move.
- A duration estimate in a violent move can be quite wrong.
- Third, applying a historical scenario to the positions held at the time rather than today's.
- Fourth, forgetting that a delta hedged book still carries gamma and vega risk.
- And fifth, the biggest one, assuming that surviving a set of scenarios proves a portfolio is safe.
- Scenarios only test the events somebody thought of, and none gives a probability.

18 Limits: guardrails on the road

- Position limit: cap on one exposure
- Scenario limit: cap on the loss in a stated stress
- Stop-loss limit: cut the position at a loss threshold
- Risk budget: a total of risk shared out among desks

IN THE LESSON

- A risk measure only helps if somebody acts on it, and that is the job of limits.
- Think of guardrails on a mountain road.
- One rail stops you straying too far to the side, another warns you of a sharp bend, and a third forces a stop.
- A position limit caps the size of an individual exposure, for example the most that can be held in one issuer, sector or country.
- A scenario limit caps the loss permitted under a specified stress.
- A stop-loss limit forces a position to be cut once losses reach a threshold.
- And risk budgeting sets a total amount of risk and divides it among desks, asset classes or managers.

19 One position, three limits

Limit used (%)

Position limit	**80%**
Scenario limit	**112%**
Stop-loss limit	**75%**

Over one hundred means the limit is breached

IN THE LESSON

- Here are the limits side by side, as bars showing how much of each limit one bond position uses.
- The vertical axis is the percentage of the limit used, so one hundred is the ceiling.
- The first bar is the position limit, at eighty per cent.
- The second is the scenario limit, and it stands highest, at one hundred twelve per cent, which means it is over the line.
- The third is the stop-loss, at seventy-five per cent.
- Read the lesson from the middle bar.
- The same position can sit comfortably inside two limits and breach a third, because each limit looks at a different property: size, stress loss or losses so far.

20 Example 7: the same position against three limits

Bond position 40m, modified duration 7

Position limit	40 ÷ 50 cap	**80%**
Scenario limit	40 × 7 × 2% = 5.6 vs 5	**112%**
Stop-loss limit	1.5 lost ÷ 2.0 allowed	**75%**
Verdict	one limit breached	**Cut or hedge**

IN THE LESSON

- Example seven.
- A trader at a made up bank holds a forty million dollar bond position with a modified duration of seven.
- The position limit allows fifty million in one issuer, so forty over fifty uses eighty per cent.
- The scenario limit says that a two hundred basis point rise in yields must not cost more than five million.
- The estimated loss is forty million times seven times two per cent, five point six million, so one hundred twelve per cent of the limit is used, a breach.
- The stop-loss allows two million of losses in the month, and one point five million have been lost, so seventy-five per cent is used.
- A position limit alone would never have caught the problem.

21 The limits' own weak spots

- Position limits ignore correlations between positions
- Scenario limits only cover scenarios someone chose
- Stop-loss can lock in a loss at the worst moment
- It may also tempt traders to gamble to recover

IN THE LESSON

- Each limit also has a weakness.
- Position limits control concentration but ignore correlations: ten positions each just under the cap, all in highly related stocks, behave like one huge bet.
- Scenario limits are only as good as the scenarios chosen.
- Stop-loss limits limit the damage, but they can crystallise a loss at the worst moment, selling at the bottom just before a recovery.
- Worse, a trader near the threshold may be tempted to take bigger risks to win the loss back before the month ends.
- Risk budgets help, but they rely on the risk measure being chosen well.
- The lesson is that no single constraint covers every risk, so firms layer them.

22 Example 8: risk budgeting in action

Total VaR budget 10m, split 5m and 5m

A: profit per 1m of VaR	4.5 ÷ 5.0	**0.9m**
B: profit per 1m of VaR	3.0 ÷ 5.0	**0.6m**
Move 1m of VaR from B to A	total VaR still 10m	**A 6m, B 4m**
Gain in expected profit	(0.9 − 0.6) × 1	**+0.3m**

IN THE LESSON

- Example eight.
- A made up fund has a risk budget of ten million of value at risk, split evenly between two managers.
- Manager A expects to earn four point five million on five million of risk, which is zero point nine million of profit per million of risk.
- Manager B expects to earn three million on the same five, zero point six per million.
- The budget is only a total, so the fund can reallocate.
- Move one million of risk from manager B to manager A, and expected profit rises by the difference, zero point three million, without adding any total risk.
- Risk budgeting asks where each unit of risk earns the most, and in practice diversification and capacity limits would stop the move going too far.

23 Capital: who gets the scarce resource

- Economic capital: absorbs large unexpected losses
- Set from VaR, stress losses or both
- Judge each desk's profit against the capital it uses
- Aim: capital where return beats the hurdle rate

IN THE LESSON

- Now to capital.
- A bank's trading desks compete for one scarce resource: the capital that would have to absorb their losses in a bad year.
- Economic capital is the amount needed to absorb large unexpected losses, and it is usually set from value at risk, from stress losses, or from both.
- Once each desk has been given capital, its profit is judged against the capital it uses, not on its own.
- Capital can be allocated by notional size, by value at risk, by stress loss or through a formal risk budget, and a good scheme reflects diversification between activities.
- The aim is to steer capital to where risk adjusted return is highest and beats a hurdle rate.

24 Profit against capital, three desks

Chart: Profit (m) vs Allocated capital (m)

- ×
- Rates 18/100 = 18%
- ×
- Equities 12/60 = 20%
- ×
- Credit 7.5/75 = 10%
- Hurdle 13%

IN THE LESSON

- Look at the picture.
- The horizontal axis is the capital allocated to each desk, in millions.
- The vertical axis is the desk's profit, in millions.
- The straight line from the origin is the thirteen per cent hurdle: any desk above the line earns more than its capital costs.
- The slope of a line from the origin to a point is that desk's return on capital.
- The equity desk makes twelve on sixty, twenty per cent, above the line.
- Rates makes eighteen on one hundred, eighteen per cent, also above.
- Credit makes seven point five on seventy-five, ten per cent, below the line.
- Rates earns the most money but not the best return.

25 Return on allocated capital

Return on capital = Profit ÷ Allocated capital

Profit	Profit earned by the desk
Allocated capital	Set from VaR or stress
Hurdle rate	Minimum return required

IN THE LESSON

- The rule for judging a desk is a short formula.
- Return on capital equals profit divided by the capital allocated to the desk.
- Then compare it with the hurdle rate, the minimum return the firm requires on its risk capital.
- If the return on capital is above the hurdle, the desk is creating value from the capital it uses.
- If it is below, the desk is consuming more than it earns, and the firm should shrink it or fix it.
- The important point is what the denominator is.
- Because the capital comes from a risk measure, such as value at risk or a stress loss, the ratio is profit per unit of risk, which is the fair way to compare desks of different size and different riskiness.

26 Example 9: ranking the desks and sharing capital

Hurdle 13%. Profit 12m, 18m, 7.5m on capital 60m, 100m, 75m

Equities	12 ÷ 60	**20%**
Rates	18 ÷ 100	**18%**
Credit	7.5 ÷ 75	**10%**
Capital by VaR share	200 × 30%, 50%, 20%	**60 / 100 / 40**

IN THE LESSON

- Example nine.
- The three desks of the made up bank earned twelve, eighteen and seven point five million on sixty million, one hundred million and seventy-five million of capital, which gives twenty, eighteen and ten per cent.
- Ranked on profit, rates wins.
- Ranked on return on capital, equities wins, and credit is below the thirteen per cent hurdle.
- Now suppose the bank has two hundred million of capital and gives it out in proportion to standalone value at risk, thirty, fifty and twenty.
- The shares are thirty, fifty and twenty per cent, so the desks get sixty million, one hundred million and forty million.
- In real life diversification between desks means the sum of the standalone figures is more than the firm's own risk.

27 Different institutions, different measures

The measure follows the way the institution can fail

Institution	Key measure
Banks	Liquidity, VaR
Asset managers	Tracking error
Hedge funds	Drawdown
DB pension funds	Surplus risk

IN THE LESSON

- Different institutions stress different measures, because each can fail in a different way.
- A bank worries about funding and borrowing, so it watches liquidity gaps and value at risk on its trading book, along with leverage, economic capital and regulatory stress tests.
- A traditional asset manager is judged against a benchmark, so its key measure is tracking error.
- A hedge fund that uses leverage and derivatives watches drawdown, the fall from its peak value, as well as value at risk and leverage.
- A defined benefit pension fund cares about the surplus, assets minus liabilities, so it measures the risk to that surplus.
- Insurers have measures of their own, which we meet in a moment.

28 Example 10: when a gain is really a loss

Assets 100, duration 5. Liabilities 96, duration 15. Yields fall 1 point

Surplus before	100 − 96	**4.0**
Assets rise	100 × 5 × 1%	**+5.0**
Liabilities rise	96 × 15 × 1%	**+14.4**
Surplus after	105 − 110.4	**−5.4**

IN THE LESSON

- Example ten.
- A defined benefit pension fund has assets of one hundred with a duration of five, and liabilities of ninety-six with a duration of fifteen, so its surplus is four.
- An asset only report would look at yields falling by one percentage point and cheer, because the bonds gain five.
- But the liabilities are a long stream of future pension payments, discounted at those same yields.
- They rise by ninety-six times fifteen times one per cent, fourteen point four.
- The surplus becomes one hundred five minus one hundred ten point four, minus five point four.
- A gain on assets turned into a deficit.
- That is why a pension fund measures surplus at risk and duration mismatch, not asset volatility alone.

29 Managers, insurers and the exam trap

- Asset managers: tracking error, active share, relative VaR
- P&C insurers: catastrophe exposure, asset-liability match
- Life insurers: duration gap and longevity risk
- Pension fund trap: assets alone miss the liabilities

IN THE LESSON

- Let us look at the other institutions briefly.
- A traditional asset manager is paid to beat a benchmark, so absolute risk is not the point.
- Tracking error measures how far its returns wander from the benchmark's, and active share measures how different its holdings are.
- A property and casualty insurer watches catastrophe exposure, such as a hurricane season, and the match between its assets and its liabilities.
- A life insurer watches the duration gap and longevity, the risk that policyholders live longer than expected.
- The exam trap is to judge a pension fund's risk by the volatility of its assets alone.
- For that fund, a low volatility portfolio can be the riskiest choice.

30 Recap: the whole toolkit

ΔV ≈ δΔS + ½Γ(ΔS)² + vega × Δσ
Return on capital = Profit ÷ Capital

Sensitivities	Beta, duration, delta, gamma, vega
Scenarios	Historical, hypothetical, reverse
Limits	Position, stress, stop, budget
Capital	Judge profit per unit of risk
The institution	Measure matches how it can fail

IN THE LESSON

- Let us pull the lesson together.
- Sensitivities are the dials: beta for shares, duration and convexity for bonds, delta, gamma and vega for options, and in the curve terms you square the move and keep the half.
- Scenarios test events: historical ones on today's positions, hypothetical ones designed on purpose, and reverse stress tests worked backwards from a loss.
- Both have weaknesses, and surviving scenarios never proves safety.
- Limits turn measures into action: position, scenario, stop-loss and a risk budget, layered because none covers everything.
- Capital goes where profit per unit of risk is highest, above the hurdle.
- And each institution picks the measures that match how it can fail.

31 Try three quick questions

1. Mod. duration 6, convexity 50, yields +2 points. Price change?
   Answer: −12% + 1% = −11%.
2. Profit 9m on capital 45m, hurdle 15%. Clears it?
   Answer: 9 ÷ 45 = 20%: yes.
3. Assets 100 dur 4, liabilities 80 dur 12, yields −1. Surplus?
   Answer: 20 + 4 − 9.6 = 14.4.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A bond portfolio has a modified duration of six and a convexity of fifty.
- Yields rise by two percentage points.
- What is the estimated price change?
- Minus twelve plus one is minus eleven per cent.
- Question two.
- A desk earns nine million on forty-five million of allocated capital, and the hurdle is fifteen per cent.
- What is its return on capital, and does it clear the hurdle?
- Nine over forty-five is twenty per cent, so yes.
- Question three.
- A pension fund has assets of one hundred with duration four, and liabilities of eighty with duration twelve.
- Yields fall by one point.
- The surplus of twenty falls to fourteen point four, so a fall in yields hurts this fund.
- Well done on finishing the questions.