CHARTA LEARNING · LESSON NOTES

Equity, volatility and rate exposure with derivatives

Derivatives and Risk Management

1 A story: the pedal on the engine

- Managers pick the shares; derivatives set the exposure
- Change risk without selling the holdings
- Futures: standard, traded on an exchange
- Forwards and swaps: tailored, over the counter

IN THE LESSON

- Think of a car with a carefully tuned engine.
- The mechanics built it with great care, and nobody wants to pull it apart every time the driver wants to go a little faster or slower.
- So the car has a pedal.
- Ashgrove Pension Fund is in the same position.
- Its equity managers have chosen shares with care, and Rania Voss does not want to disturb them just because the investment committee has changed its view on markets.
- She needs a pedal.
- Equity derivatives are that pedal.
- A future, a forward or a swap changes how much the fund gains or loses when the market moves, while the shares stay exactly where they are.

2 How many futures contracts?

Contracts = ((β target − β now) ÷ β futures) × (Value ÷ (Price × Multiplier))

Beta gap	Target minus current beta
Futures beta	Close to one for an index
Portfolio value	The money being adjusted
Contract value	Price times multiplier

IN THE LESSON

- Here is the sizing rule.
- Rania Voss starts with the beta of her equity portfolio, which says how much it moves when the market moves by one.
- She subtracts that from the beta she wants.
- That is the beta gap.
- She divides by the beta of the futures contract, which is close to one for an index future.
- Then she multiplies by how many contracts' worth of money she is adjusting: the portfolio value divided by the contract value.
- The contract value is the futures price times the multiplier, the money each index point is worth.
- A negative answer means sell and a positive answer means buy.
- Forget the multiplier and the answer is wildly wrong.

3 Example 1: cut the beta

Equity sleeve 100m, beta 1.10; target 0.80; futures 4,000

Contract value	4,000 × 250	**1,000,000**
Beta gap	0.80 − 1.10	**−0.30**
Contracts	−0.30 ÷ 1.0 × 100m ÷ 1m	**Sell 30**
Exposure removed	30 × 1m	**30m**
Trap: forget the multiplier	−0.30 × 100m ÷ 4,000	**−7,500**

IN THE LESSON

- Now Ashgrove.
- The committee is nervous for a quarter.
- Its domestic equity sleeve is worth one hundred million, with a beta of one point one, and it wants a beta of point eight.
- Index futures stand at four thousand, with a multiplier of two hundred and fifty, so each contract is worth one million.
- The beta gap is point eight minus one point one, which is minus point three.
- Divide by the futures beta of one, then multiply by one hundred million over one million.
- The answer is minus thirty.
- Sell thirty futures.
- That removes thirty million of market exposure, exactly point three of one hundred million.
- Now the trap.
- Leave out the multiplier and divide by four thousand, and you get seven thousand five hundred contracts, two hundred and fifty times too many.

4 Example 2: equitise the cash

Cash 20m, target beta 1.0; index +3%; cash earns 1%

Contracts to buy	(1.0 − 0) ÷ 1.0 × 20m ÷ 1m	**Buy 20**
Futures gain	20 × 1m × 3%	**600,000**
Cash interest	20m × 1%	**200,000**
Total gain	600,000 + 200,000	**800,000**
Return on the 20m	800,000 ÷ 20m	**4.0%**

IN THE LESSON

- Futures also work in the other direction.
- Suppose Ashgrove has twenty million of cash waiting to be invested, and wants it to behave like equities, a beta of one.
- Buying twenty futures, each worth one million, does exactly that.
- This is called equitising cash.
- Over the quarter the index rises three per cent.
- The futures gain point six million, which is twenty contracts times one million times three per cent.
- But the cash was never spent, so it keeps earning, say one per cent for the quarter, which is point two million.
- The total is point eight million, four per cent of the twenty million.
- The futures delivered the index return, and the cash supplied the short term rate on top.
- If the cash is spent elsewhere, the match breaks.

5 Equity swap: exposure out, shares kept

Shareholder

Keeps shares and votes → Pays the index return → Receives floating rate

Dealer

Holds no shares → Receives the index return → Pays floating, covers losses

IN THE LESSON

- Now the swap, for a holder who cannot or will not sell the shares.
- Perhaps selling would trigger a tax bill, or the shares carry votes that matter.
- Follow the top row.
- The shareholder keeps the shares, and with them the votes.
- Each period the shareholder pays the dealer the return on the index, and receives a floating rate in exchange.
- Now the bottom row.
- The dealer holds no shares, yet receives the return on the index, so the dealer has the exposure without buying anything.
- Notice what happens when shares fall.
- The index return is negative, and the payer of the equity return receives that loss from the dealer.
- So the shareholder has handed the market risk away, and ends up with roughly the floating rate.
- Each side now relies on the other to pay.

6 Example 3: the swap in numbers

Shares 50m; swap 50m; index −8%; floating 1% a quarter

Shares lose	50m × −8%	**−4.0m**
Equity leg: payer receives loss	pay −8%	**+4.0m**
Floating leg received	50m × 1%	**+0.5m**
Net result	−4.0 + 4.0 + 0.5	**+0.5m**
Same swap if index +6%	3.0 − 3.0 + 0.5	**+0.5m**

IN THE LESSON

- Here are the numbers.
- Ashgrove holds fifty million of shares that it wants to hedge with a swap of the same size.
- In a bad quarter the index falls eight per cent.
- The shares lose four million.
- On the swap, Ashgrove pays the index return, which is minus eight per cent, so in fact it receives four million from the dealer.
- It also receives the floating leg, one per cent of fifty million, which is point five million.
- The net result is plus point five million.
- Now suppose the index had risen six per cent.
- The shares gain three million, the swap takes three million away, and the floating leg again adds point five.
- Same net result.
- Up or down, Ashgrove earns the floating rate and carries no market exposure.
- The swap has traded the exposure for a rate.

7 Futures, forwards and swaps compared

Three tools, three traits

Tool	Key trait
Equity futures	Daily margin
Equity forward	Tailored
Equity swap	Keeps shares
Forward or swap	Default risk

IN THE LESSON

- A quick comparison.
- Equity futures are standard contracts on an exchange.
- They are marked to market every day and backed by margin, so counterparty risk is small, but the terms cannot be bent.
- Equity forwards are the tailored cousin, written over the counter for the size and date you want, usually settled once at the end.
- Equity swaps exchange returns period after period, and, as we saw, let the holder keep the shares.
- Forwards and swaps share one weakness.
- If the other side fails to pay, Ashgrove loses what it was owed, so counterparty risk needs managing with collateral and limits.
- And there is a quiet assumption inside our sizing rule: that the portfolio's beta stays where it was measured, which it rarely does perfectly.

8 A story: the alarm that pays when it rings

- Volatility: how violently prices swing
- It jumps when stocks fall
- A long volatility position can cushion the loss
- Variance swaps and futures trade it directly

IN THE LESSON

- Imagine a smoke alarm that did not only warn you of fire, but paid you when it rang.
- Volatility is like that for investors.
- Volatility measures how violently prices swing.
- It is quiet in calm markets, and it jumps when markets fall, because fear makes prices lurch.
- That is the useful fact for Rania Voss.
- Her fund loses when shares fall, and volatility tends to rise at just that moment.
- So a position that gains when volatility rises can cushion the loss.
- Options give some volatility exposure, but they also carry the direction of the market.
- Variance swaps and volatility futures let her trade volatility more purely.

9 What a variance swap pays

Variance notional = vega notional ÷ (2 × strike vol)
Payoff = variance notional × (realised variance − strike variance)

Vega notional	Gain per volatility point
Strike volatility	Agreed on day one
Variance notional	Scales the payoff
Realised variance	Realised volatility squared

IN THE LESSON

- A variance swap pays on how much the market actually moved.
- Two parties agree a strike, a level of volatility.
- At the end, the buyer receives the realised variance minus the strike variance, times the variance notional.
- Variance is simply volatility squared.
- The variance notional is set so that the swap feels like a bet of a fixed amount per volatility point.
- That fixed amount is the vega notional, and the variance notional is the vega notional divided by two times the strike volatility.
- Why the two?
- Because a small change in volatility changes variance by about two times the volatility.
- The division cancels that out, so near the strike one volatility point pays about the vega notional.

10 Example 4: a variance swap pays

Vega notional 1,000,000; strike 20; realised volatility 25

Variance notional	1,000,000 ÷ (2 × 20)	**25,000**
Strike variance	20^2	**400**
Realised variance	25^2	**625**
Variance swap payoff	25,000 × (625 − 400)	**5,625,000**
A linear bet pays	1,000,000 × (25 − 20)	**5,000,000**

IN THE LESSON

- Ashgrove buys a variance swap with a vega notional of one million and a strike of twenty.
- The variance notional is one million divided by two times twenty, which is twenty five thousand.
- The strike variance is twenty squared, four hundred.
- After the year, realised volatility turns out to be twenty five, so realised variance is six hundred and twenty five.
- The gap is two hundred and twenty five variance points, times twenty five thousand, giving five million six hundred and twenty five thousand.
- Compare that with a simple bet of one million per volatility point.
- Five points above the strike would pay five million.
- The variance swap paid more.
- That extra comes from squaring, and it is what the next picture shows.

11 A payoff that bends upward

Chart: Payoff (millions) vs Realised volatility (10 to 30)

- Strike 20: both pay zero
- Down at 10: −7.5 against −10
- Variance swap
- Linear bet
- At 25: 5.625 against 5.0

IN THE LESSON

- Here is that extra, drawn.
- The horizontal axis is realised volatility, from ten to thirty.
- The vertical axis is the payoff in millions.
- Start with the flat line at zero, where nothing is paid.
- The straight rising grey line is the plain bet of one million per point.
- It crosses zero at the strike of twenty.
- Now the curve.
- It is the variance swap, and it also crosses zero at twenty, where the two agree.
- Above the strike, the curve bends upward and pays more than the line.
- At twenty five it pays five point six two five million against five.
- Below the strike, the curve loses less than the line: at ten it loses seven and a half million, not ten.
- The payoff is convex.
- For a buyer, big volatility moves pay disproportionately well.

12 Example 5: value before expiry

4 months in: realised 25; next 8 months implied 22; DF 0.99

Expected variance	1/3 × 625 + 2/3 × 484	**531**
Gap to strike variance	531 − 400	**131**
Before discounting	25,000 × 131	**3,275,000**
Value, factor 0.99	3,275,000 × 0.99	**3,242,250**

IN THE LESSON

- Now value the swap before it ends.
- The rule is simple.
- Expected variance is a blend of what has already happened and what the market expects for the rest, weighted by time.
- Implied volatility is the volatility that option prices suggest.
- Four months into the year, Ashgrove's swap has realised twenty five so far, and implied volatility for the remaining eight months is twenty two.
- Four months is one third of the year and eight months is two thirds.
- One third of six hundred and twenty five, plus two thirds of four hundred and eighty four, is five hundred and thirty one.
- The gap to the strike variance is one hundred and thirty one.
- Times twenty five thousand gives three million two hundred and seventy five thousand.
- With a discount factor of point nine nine, the value is three million two hundred and forty two thousand, two hundred and fifty.
- That is what the swap is worth today.

13 Volatility futures: hedge and carry

Sell off

Stocks fall sharply → Volatility spikes → Long futures gain

Calm market

Markets stay calm → Futures drift to spot → Each roll loses carry

IN THE LESSON

- Variance swaps are over the counter.
- A simpler tool is a future on a volatility index, a published measure of the volatility that options imply.
- Follow the top row.
- Stocks fall sharply, volatility spikes, and a long position in volatility futures gains, just when the equity portfolio is losing.
- That is the hedge.
- Now the bottom row, which is the catch.
- In calm markets these futures usually trade above the index level, a pattern called contango.
- As a future ages, its price drifts down towards the index.
- A fund that holds the future long and keeps rolling it into the next one loses a little each time, even if nothing happens.
- So the hedge costs money when markets are calm.

14 Example 6: what the carry costs

Long 200 volatility futures; each point worth 1,000

Future against index	18 against 16	**2 points**
Loss per roll, calm market	2 × 1,000 × 200	**400,000**
Six monthly rolls	6 × 400,000	**2,400,000**
Drag on 200m of equities	2.4m ÷ 200m	**1.2%**

IN THE LESSON

- Here is what that cost looks like.
- Ashgrove holds two hundred long futures on a volatility index, and each index point is worth one thousand.
- The future costs eighteen while the index itself sits at sixteen, so the future is two points above.
- If markets stay calm, the future falls to sixteen by expiry.
- The loss is two times one thousand times two hundred, which is four hundred thousand a roll.
- Roll monthly for six months and the loss is two million four hundred thousand.
- Against a two hundred million equity portfolio, that is one point two per cent, paid for protection that was never used.
- This is an illustration with made up numbers, but the pattern is real.
- Buy the hedge when you need it, and size it knowing it leaks.

15 A story: two sliders on a mixing desk

- Two sliders: equity risk and interest rate risk
- Measure now, set the target, find the gap
- Size a derivative to close the gap exactly
- Beta measures equity risk; BPV measures rate risk

IN THE LESSON

- Think of a mixing desk with two sliders.
- One slider sets how much equity risk a fund carries.
- The other sets how much interest rate risk.
- A sound engineer does not rebuild the band to move a slider.
- She just moves it.
- Rania Voss wants the same control for Ashgrove.
- The method is the same for both sliders.
- First measure where the exposure is now.
- Then decide where the target is.
- The difference is the gap.
- Then size a derivative position that closes that gap exactly, and no more.
- Equity risk is measured with beta.
- Interest rate risk is measured with basis point value, which people call B P V, the money the bonds gain or lose when rates move by a tiny step called a basis point.

16 Sizing both legs

Equity contracts = (β gap ÷ β futures) × (Value ÷ Contract value)
Rate contracts = (BPV target − BPV now) ÷ BPV futures

Beta gap	Target beta minus current
Contract value	Price times multiplier
BPV gap	Target BPV minus current
Futures BPV	Change per basis point

IN THE LESSON

- Two formulas, one for each slider.
- The equity one you know already: the beta gap, divided by the futures beta, times the portfolio value over the contract value.
- The rates one is simpler.
- Take the target basis point value, subtract the current one, and divide by the basis point value of one bond futures contract.
- A positive answer means buy futures, and a negative answer means sell.
- Why does selling bond futures cut the bond portfolio's sensitivity?
- Because a bond future gains when rates fall, just like bonds, so selling it adds an opposite exposure.
- The physical bonds are untouched.
- Only the net sensitivity changes.

17 Example 7: set both targets

Equity 200m β 0.90 → 1.10; bonds 150m, BPV 120,000 → 90,000

Index futures contract	3,200 × 250	**800,000**
Equity leg	0.20 × 200m ÷ 800,000	**Buy 50**
Rates leg	(90,000 − 120,000) ÷ 75	**Sell 400**
Equity exposure added	50 × 800,000	**40m**
BPV removed	400 × 75	**30,000**

IN THE LESSON

- Rania Voss has a new set of targets.
- Ashgrove's international equity sleeve holds two hundred million at a beta of point nine, and the committee now wants one point one.
- At the same time, its bond holding of one hundred and fifty million has a basis point value of one hundred and twenty thousand, and the committee wants it cut to ninety thousand.
- Index futures stand at three thousand two hundred, with a multiplier of two hundred and fifty, worth eight hundred thousand each.
- For the equity leg, point two times two hundred million divided by eight hundred thousand is fifty.
- Buy fifty.
- For the rates leg, ninety thousand minus one hundred and twenty thousand, divided by a futures basis point value of seventy five, is minus four hundred.
- Sell four hundred.
- Fifty contracts add forty million of exposure, and four hundred remove thirty thousand of basis point value.

18 Two dials, two legs

Equity dial

Beta 0.90 on 200m → Buy 50 index futures → Beta 1.10, shares kept

Rates dial

BPV 120,000 on bonds → Sell 400 bond futures → BPV 90,000, bonds kept

IN THE LESSON

- Here are the same two legs side by side.
- Top row, the equity dial.
- The sleeve starts at a beta of point nine on two hundred million.
- Fifty index futures are bought.
- The beta becomes one point one, and the shares are untouched.
- Bottom row, the rates dial.
- The bonds start with a basis point value of one hundred and twenty thousand.
- Four hundred bond futures are sold.
- The value falls to ninety thousand, and the bonds stay where they are.
- The two legs are independent.
- Ashgrove could have raised both, cut both, or moved just one.
- And notice what did not happen.
- No bond and no share changed hands, so the managers running the physical portfolios carry on as before.

19 Choosing the tool for the target

If Ashgrove wants this, it uses that

Ashgrove wants	Tool
Quick, cheap, whole fund	Futures
Keep shares, maybe defer tax	Swap
Floor losses, keep upside	Puts
Guard against rising rates	Swaption

IN THE LESSON

- Futures are not the only way.
- Rania Voss chooses the tool by the shape of the target.
- If the committee wants a quick, cheap move across the whole fund, futures are best.
- If Ashgrove must keep the shares for tax or voting reasons, an equity swap does it.
- A total return swap that receives the index adds equity exposure without buying a share, and a swap that pays fixed cuts interest rate exposure.
- If the target is one sided, say a firm limit on losses but with the gains kept, options fit better.
- Index puts set a floor under equities, and payer swaptions guard against rates rising.
- Each choice has a price, and the next example shows one.

20 Example 8: a floor with index puts

Equity 200m; floor at −10%; premium 2.5% of the portfolio

Loss before the floor	10% × 200m	**20.0m**
Premium paid	2.5% × 200m	**5.0m**
Worst case, any fall	20.0 + 5.0	**25.0m**
Equities +15% with puts	30.0 − 5.0	**+25.0m**
Equities +15%, full futures hedge	30 gained, 30 paid	**0.0m**

IN THE LESSON

- Here is the one sided case.
- Ashgrove holds two hundred million of equities and the committee will not accept a fall of more than ten per cent.
- Rania Voss buys index puts that start paying at a ten per cent fall, and the premium is two and a half per cent of the portfolio.
- The loss before the floor bites is twenty million.
- Add five million of premium, and the worst case, however far markets fall, is twenty five million.
- Now suppose equities rise fifteen per cent.
- The shares gain thirty million, less the five million premium, so plus twenty five million.
- A full futures hedge would have given up the whole thirty million gain.
- The puts cost five million for keeping that upside.
- That is the trade off, and it is a decision the committee must defend.

21 Mistakes to avoid so far

- Leaving out the contract multiplier
- Trusting a beta that drifts over time
- Ignoring counterparty risk in forwards and swaps
- Holding long volatility futures through contango
- Futures on cash that is not earning the short rate

IN THE LESSON

- Let us collect the common mistakes so far.
- One: leaving out the contract multiplier, which makes the contract count wildly wrong.
- Two: trusting a beta that may drift, so the hedge slowly stops matching the portfolio.
- Three: forgetting the counterparty in a forward or a swap, because the other side might not pay.
- Four: holding long volatility futures through calm markets, where contango drains value on every roll.
- Five: buying futures against idle cash that is not left earning the short term rate, so the match to the index breaks.
- Keep these five in mind, because the exam loves each of them.

22 A story: the rudder and the weather vane

- Policy mix 60% equities, 40% bonds, drifts
- Futures restore the weights in an afternoon
- Physical trades wait for the scheduled rebalance
- Prices also carry the market's expectations

IN THE LESSON

- Imagine steering a ship by turning a rudder, rather than rebuilding the engine room.
- A rebalancing overlay is that rudder.
- Ashgrove has a policy mix of sixty per cent equities and forty per cent bonds, set by the trustees.
- But markets move, and the mix drifts.
- Selling shares and buying bonds in the real portfolios takes time, costs money and disturbs managers who are partway through their own plans.
- Futures can restore the policy weights in an afternoon, and the physical trades can wait for the scheduled rebalance.
- The second job is quieter.
- Derivative prices are set by thousands of traders, so they contain opinions about the future, and anyone can read them.

23 Example 9: rebalance with futures

Fund 1,000m, policy 60/40, now 64/36, bond duration 6

Equity overweight	(64% − 60%) × 1,000m	**40m**
Equity futures to sell	40m ÷ 1,000,000	**Sell 40**
BPV to add	40m × 6 × 0.0001	**24,000**
Bond futures to buy	24,000 ÷ 75	**Buy 320**
Physical trades	managers keep holdings	**none**

IN THE LESSON

- A rally has pushed Ashgrove, a fund of one thousand million, from sixty forty to sixty four and thirty six.
- The next scheduled rebalance is still weeks away.
- The overweight in equities is four per cent of one thousand million, which is forty million.
- Each equity future is worth one million, so sell forty.
- To add forty million of bonds, with a duration of six, the basis point value needed is forty million times six times point zero zero zero one, which is twenty four thousand.
- A bond future has a basis point value of seventy five, so buy three hundred and twenty.
- And the physical trades?
- None.
- The managers keep their holdings, and the overlay is closed out when the real rebalance happens.

24 Back on target, no shares sold

Share of fund (%)

Equity: drifted	**64%**
Equity: overlay	**60%**
Bonds: drifted	**36%**
Bonds: overlay	**40%**

Policy mix: 60% equities, 40% bonds

IN THE LESSON

- Here is the result.
- The vertical axis is the share of the fund, in per cent.
- The first bar is the equity weight after the rally, sixty four.
- The second is the equity weight with the overlay, sixty, back on target.
- The third and fourth bars are bonds.
- After the rally they hold thirty six per cent, and with the overlay they are back to forty.
- Notice that the overlay changes the exposures that matter for risk, not the securities.
- The cost is that the fund now carries a futures position, with margin to post and a small gap between the futures and the actual holdings, called tracking risk.
- That is the price of doing the job quickly.

25 Reading a rate from a futures price

Implied rate = 100 − futures price
Probability = (implied − current) ÷ (after move − current)

Implied rate	100 minus the price
Current rate	Policy rate today
Rate after move	Current plus move size
Probability	Chance of the move

IN THE LESSON

- Now the second job: reading the market's expectations.
- A policy rate future is quoted as one hundred minus the expected average policy rate.
- So a higher price means a lower expected rate.
- Next, compare that with today's rate.
- Suppose the central bank can either leave the rate alone or raise it by a fixed step.
- The implied rate is then a weighted average of those two outcomes, and the weight on the rise is the probability.
- Rearranged, the probability is the implied rate minus the current rate, divided by the rate after the move minus the current rate.
- It is the share of the way the price has travelled from no change to a full move.

26 Example 10: the chance of a rise

Policy rate 3.50%; future 96.40; possible move +0.25

Implied rate	100 − 96.40	**3.60%**
Priced in above today	3.60 − 3.50	**0.10**
Size of a full move	3.75 − 3.50	**0.25**
Probability of a rise	0.10 ÷ 0.25	**40%**
Probability of no change	1 − 0.40	**60%**

IN THE LESSON

- Rania Voss puts numbers in.
- The policy rate is three point five zero per cent and the future is priced at ninety six point four zero.
- The implied rate is one hundred minus ninety six point four zero, which is three point six zero.
- That is point one zero above today.
- If the only possible move is a rise of twenty five basis points, to three point seven five, the full step is point two five.
- The probability of a rise is point one zero divided by point two five, which is forty per cent.
- The probability of no change is the rest, sixty per cent.
- Notice the answer depends entirely on that assumption about the size of the move.

27 How far along the move is priced

- Now 3.50%
- Implied 3.60%, 40% along
- After rise 3.75%

IN THE LESSON

- Here is the same idea on a line.
- The line runs from no change, on the left, to a rise on the right.
- The first mark is today's rate, three point five zero per cent.
- The last mark is the rate after a full rise, three point seven five.
- The middle mark is the rate implied by the futures price, three point six zero.
- See how far along it sits, a little under half of the way from left to right.
- That distance is the probability, forty per cent.
- If the price moved to the right hand mark, the market would be certain of the rise.
- If it sat on the left hand mark, the market would expect nothing at all.

28 Example 11: expected inflation

Ashgrove pays fixed, receives inflation, on 100m

Expected inflation (fixed rate)		**2.5%**
Inflation turns out at		**3.2%**
Receive less pay	3.2% − 2.5%	**0.7%**
Net per year on 100m	0.7% × 100m	**0.7m**
Reading the fixed rate	may include a premium	**caution**

IN THE LESSON

- Prices also reveal expected inflation.
- In an inflation swap, one side pays a fixed rate and the other pays the inflation that actually occurs.
- The fixed rate that makes the swap fair at the start is the market's expected inflation.
- Suppose it is two and a half per cent.
- Ashgrove has liabilities that rise with inflation, so it pays fixed and receives realised inflation on a notional of one hundred million.
- If inflation turns out to be three point two per cent, it receives point seven per cent more than it pays, which is seven hundred thousand for the year.
- But be careful in reading the fixed rate.
- It may include a premium that investors demand for bearing inflation risk, so it is the market's price, not a pure forecast.

29 Traps when reading prices

- Assuming only one size of move is possible
- Prices include risk premia, not a pure forecast
- Implied rates average over the contract's life
- An overlay is a bridge: margin, tracking, rebalance later

IN THE LESSON

- Reading prices takes care.
- First trap: assuming only one size of move is possible.
- If the market is also pricing a larger move, or a cut, the simple probability is wrong.
- Second trap: forgetting risk premia.
- Prices are what investors will pay, and that includes compensation for bearing risk, so an implied probability is not a pure forecast.
- Third, a rate future prices the average rate over its life, so the date of the meeting inside that period matters.
- And a last point about overlays: they are a bridge.
- Margin must be posted, the futures can drift from the holdings, and the real rebalance still has to happen later.

30 Recap: the lesson in five lines

- Futures: beta gap × value ÷ contract value
- Variance swap: convex; futures leak carry
- Targets: size the equity leg and the BPV leg
- Overlays rebalance without touching managers
- Prices show expectations, with risk premia

IN THE LESSON

- Let us pull the lesson together.
- First, futures size by beta gap times value over contract value, and forwards and swaps do the same job over the counter, with swaps letting the holder keep the shares.
- Second, a variance swap pays variance notional times realised variance less strike variance, so its payoff is convex, and volatility futures hedge sell offs but leak carry.
- Third, to set targets, measure the gap and size each leg, one for beta and one for basis point value.
- Fourth, futures overlays rebalance in an afternoon without touching the managers.
- Fifth, prices reveal expectations, but only if you respect the assumptions and the risk premia.

31 Try three quick questions

1. Beta 1.2 to 0.9; 50m of shares; futures 500,000, beta 1.0. Contracts?
   Answer: (0.9 − 1.2) ÷ 1.0 × 50m ÷ 500,000 = −30. Sell 30.
2. Vega notional 400,000; strike 20; realised 22. Payoff?
   Answer: 10,000 × (484 − 400) = 840,000.
3. Rate 1.80%; future 98.00; move 0.25. Probability of a rise?
   Answer: (2.00 − 1.80) ÷ 0.25 = 80%.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A fund holds fifty million of shares at a beta of one point two and wants point nine.
- Futures are worth half a million each, with a beta of one.
- How many contracts?
- Minus point three times one hundred is minus thirty, so sell thirty.
- Question two.
- A variance swap has a vega notional of four hundred thousand and a strike of twenty.
- Realised volatility is twenty two.
- What does it pay?
- The variance notional is ten thousand, times eighty four, so it pays eight hundred and forty thousand.
- Question three.
- The current policy rate is one point eight zero per cent and the future is priced at ninety eight.
- A move would be twenty five basis points.
- What is the probability of a rise?
- The implied rate is two, so point two over point two five gives eighty per cent, a fairly confident rise.