Option combinations

Derivatives

1 A story: the copy shop

- Some investors cannot hold the share
- Options plus a bond can copy it exactly
- The share must pay no income before expiry
- The link between the pieces is put-call parity

IN THE LESSON

- Imagine you want a particular chair but the shop will not sell it to you this month.
- A good carpenter says, I cannot sell you that chair, but I can build one from the same parts, and it will sit exactly like the original.
- Options let a portfolio manager do the same with an investment.
- Some funds cannot hold a share because of a rule, a delay or a cost.
- Others find the options cheaper or quicker to trade.
- Dev Rahman wants to know whether a long call, a short put and a safe bond can behave exactly like one Tarn Foods share.
- They can, provided the share pays no income, and the reason is called put call parity.

2 Two options, then a bond

Step one: the forward

Long call → Short put → Synthetic forward

Step two: the share

Synthetic forward → Bond paying X → Synthetic share

IN THE LESSON

- Here is the recipe as a picture.
- Read the top row first.
- A long call and a short put, both with the same strike and the same expiry, are bought and sold together.
- If the share ends above the strike, the call pays the difference and the put expires worthless.
- If the share ends below, the call is worthless, and you owe the shortfall on the put you wrote.
- Either way you receive the final price minus the strike.
- That is a synthetic forward.
- Now the bottom row.
- The forward is missing the strike amount, so we add a bond that pays exactly the strike at expiry.
- Forward plus bond gives the share itself.

3 Put-call parity

S + p = c + PV(X)
S = c − p + PV(X)

Share price	What the share costs today
Put premium	Same strike, same expiry
Call premium	Same strike, same expiry
Present value of X	Bond that pays X at expiry

IN THE LESSON

- Now the same recipe as an equation.
- Put call parity says a share plus a put costs the same as a call plus the present value of the strike.
- Here the call and the put have the same strike and expiry, and the strike is written capital X.
- The present value of the strike is what a safe bond paying X at expiry costs today.
- Rearrange it and you get the share on one side: the share equals the call, minus the put, plus the bond.
- That is the recipe we just drew.
- If someone offers a call, a put and a bond that do not satisfy the equation, a trader can buy the cheap side and sell the dear side and lock in a profit.

4 Example 1: does the copy cost the same?

Share 50, no income; strike 52; 1 year; rate 4%; call 5, put 5

Bond paying 52	52 ÷ 1.04	**50.00**
Call bought, put sold	5.00 − 5.00	**0.00**
Cost of the package	0.00 + 50.00	**50.00**
Cost of the share	price today	**50.00**
Verdict	the copy costs the same	**Parity holds**

IN THE LESSON

- Let us test it on Tarn Foods.
- The share is fifty.
- Dev Rahman looks at a one year call and a one year put, both struck at fifty two, each costing five.
- The risk free rate is four per cent.
- The present value of the strike is fifty two divided by one point zero four, which is exactly fifty.
- Now price the package.
- Buy the call for five, sell the put for five, and the options net to nothing.
- Add the bond for fifty, and the whole package costs fifty.
- That is the price of the share.
- The market is consistent, and the copy costs exactly what the original costs.
- If the package had cost more, buying the share itself would be the better deal.

5 Example 2: the copy pays the same

Call minus put plus bond, strike 52

Share ends at 40	0 − 12 + 52	**40**
Share ends at 52	0 − 0 + 52	**52**
Share ends at 65	13 − 0 + 52	**65**
Verdict	equals the share each time	**Same payoff**

IN THE LESSON

- Same cost is only half the proof.
- The copy must also pay the same at expiry.
- Check three prices for Tarn Foods.
- If the share ends at forty, the call is worthless, the put you wrote costs you twelve, and the bond pays fifty two.
- Zero minus twelve plus fifty two is forty, the share price.
- If the share ends at fifty two, nothing is owed on either option, and the bond pays fifty two.
- If it ends at sixty five, the call pays thirteen, the put is worthless, and the bond pays fifty two, which adds to sixty five.
- At every price, the package is worth exactly what the share is worth.

6 Example 3: a synthetic put

Synthetic put: long call, short share, bond paying 52

Share ends at 40	0 − 40 + 52	**12**
Share ends at 65	13 − 65 + 52	**0**
Real 52 put pays at 40	52 − 40	**12**
Real 52 put pays at 65	nothing owed	**0**
Verdict	the same payoffs	**Put copied**

IN THE LESSON

- The same algebra builds other things.
- Move the terms of the equation around and a put equals a call, minus the share, plus the bond.
- So a manager who wants the protection of a put, but finds the real one expensive or hard to trade, can build it.
- Buy the call, sell the share short, and buy the bond.
- Check it with the same prices.
- If the share ends at forty, the call pays nothing, the short sale loses forty, and the bond pays fifty two, which leaves twelve.
- A real put struck at fifty two pays twelve.
- If the share ends at sixty five, the call pays thirteen, the short share costs sixty five, the bond pays fifty two, and the total is nothing, just like the put.

7 The same parts, rearranged

Four things you can build, with no income on the share

Position	Recipe
Synthetic forward	Call, short put
Synthetic share	Forward, bond
Synthetic put	Call, short share, bond
Synthetic call	Share, put, loan

IN THE LESSON

- Here is the family of synthetic positions in one table.
- A long call plus a short put is a synthetic forward, and adding the bond turns it into a synthetic share.
- A long call, a short share and the bond make a synthetic put.
- And the share, a long put, and borrowing the present value of the strike make a synthetic call.
- Notice that every recipe has the same shape: options, then something that carries the strike, either a bond you hold or a loan you owe.
- Managers use these when the options are cheaper or quicker than the real thing, or when a rule blocks the direct holding.
- The slip to avoid is leaving out the bond.

8 A story: the sign on the garden gate

- Own the shares, then sell a call on them
- The premium is paid to you today
- You promise to sell at the strike if asked
- Covered: the shares are in hand to deliver
- Nora would happily sell Tarn Foods at 55

IN THE LESSON

- Picture a gardener who owns an apple tree and has a price in mind.
- She tells a neighbour, I will sell you my crop at a price I have chosen, and I want paying now for the promise.
- If the apples sell for less, she keeps the payment.
- If they sell for more, she still has to hand them over at the chosen price.
- That is a covered call.
- An investor already owns the shares, then writes a call on them.
- Writing means selling the call and receiving its premium.
- The word covered means the shares are in hand, so the investor can always deliver.
- Nora Ashcombe likes the idea, because at fifty five she would happily sell Tarn Foods.

9 Why would anyone write one?

Nora holds 10,000 Tarn Foods shares bought at 50

Goal	How the call helps
Earn income	Premium: 2
Sell at a target	Sold at 55
Lift a flat market	Return plus 2
Hedge a fall (not a goal)	Only 2 of cushion

IN THE LESSON

- There are three honest reasons to write a covered call.
- The first is income.
- The premium arrives today and is kept whatever happens.
- For Nora, the call struck at fifty five pays two a share.
- The second is selling at a target.
- If the price climbs to fifty five or more, the shares are called away at fifty five, which is the price she already said she would accept.
- The third is a modest cushion in a market that goes nowhere: the premium adds two to the return of a flat share.
- What the call is not is protection.
- The premium only softens a fall by two, as we will see in the numbers.

10 How a covered call is built

Nora's covered call: 3 months, 20,000 premium

Own 10,000 shares at 50 → Write 55 call for 2 → Wait for expiry

IN THE LESSON

- The construction has only two moves.
- First, Nora owns ten thousand Tarn Foods shares, bought at fifty.
- Second, Dev Rahman writes ten thousand three month calls on those shares, with a strike of fifty five, and collects two a share.
- That is twenty thousand in premium, in her account on day one.
- Then we wait for expiry.
- If the share finishes above fifty five, the buyer exercises, Nora delivers at fifty five and her gain stops there.
- If it finishes below, the call lapses worthless, she keeps the shares, and she keeps the premium too.
- Either way, the premium has been earned.

11 Covered call: payoff and profit

Value at expiry = S − max(0, S − X); profit = value − S0 + c
Max profit = X − S0 + c; max loss = breakeven = S0 − c

Value at expiry	Flat at X once S passes X
Profit	Value, less cost, plus premium
Maximum profit	Reached at X or above
Breakeven	Share price where profit is zero

IN THE LESSON

- Now the arithmetic.
- At expiry, the covered call is worth the share price minus whatever the call costs the writer.
- The call costs the writer the amount by which the price exceeds the strike, or nothing if it does not.
- So above the strike the value is flat at the strike.
- Profit then takes the value, subtracts the price originally paid for the shares, and adds the premium received.
- The most she can make is the strike minus the purchase price plus the premium.
- The worst case is the share becoming worthless, so she loses the purchase price less the premium.
- And breakeven, the share price where profit is zero, is the purchase price less the premium.

12 Example 4: the best, the worst, the breakeven

Cost 50; sell a 55 call for 2; 10,000 shares

Maximum profit	55 − 50 + 2 = 7 a share	**70,000**
Breakeven price	50 − 2	**48**
Maximum loss	50 − 2 = 48 a share	**480,000**
Premium kept in every case	2 × 10,000	**20,000**

IN THE LESSON

- Put Nora's numbers in.
- The shares cost fifty, the strike is fifty five, the premium is two.
- The most she can make is fifty five minus fifty plus two, which is seven a share, or seventy thousand on ten thousand shares.
- The five is the rise up to the strike, and the two is the premium.
- The breakeven is fifty minus two, forty eight.
- The worst case needs the share to become worthless.
- She would lose fifty a share, less the two she received, forty eight, so four hundred and eighty thousand.
- That is not far from the loss of holding the shares alone.
- The call cushions a fall by only the premium.

13 The covered call on a graph

Chart: Profit per share vs Share price at expiry (30 to 70)

- Breakeven 48
- Capped at 7 above 55
- Share alone

IN THE LESSON

- Here is the covered call as a picture.
- The horizontal axis is the share price at expiry, running from thirty on the left to seventy on the right, and the vertical axis is profit per share.
- The dashed grey line is zero profit.
- The thin line that keeps climbing is the share held alone, rising one for one with the price.
- The thick line is the covered call.
- It runs above the share alone, thanks to the premium, until the price reaches the strike of fifty five.
- Then it goes flat at seven, because every point above the strike belongs to the call buyer.
- It crosses zero at forty eight, the breakeven.
- So the picture shows income earned, and upside given away.

14 Example 5: four prices at expiry

Cost 50; strike 55; premium 2

Share ends at 40	40 − 50 + 2	**−8**
Share ends at 50	50 − 50 + 2	**2**
Share ends at 55	55 − 50 + 2	**7**
Share ends at 65	55 − 50 + 2	**7**
Shares alone at 65	65 − 50	**15**

IN THE LESSON

- Now follow four possible endings for Tarn Foods.
- If the share falls to forty, the call is worthless, and the profit is forty minus fifty plus two, a loss of eight.
- The shares alone would have lost more, so the call helped by exactly its premium.
- If the share ends at fifty, the profit is the two of premium and nothing else.
- At fifty five the call is only just worthless, and the profit is the full seven.
- If the share climbs to sixty five, the call buyer takes the rise above the strike.
- Nora still makes only seven, while the shares alone would have made fifteen.
- That difference is the price of the cap.

15 A story: insurance on the house

- Own the shares, then buy a put on them
- The strike is the floor under the value
- The premium is the price of the policy
- Purchase price minus strike is the deductible
- Nora: a 45 put costing 1.50

IN THE LESSON

- When you insure a house, you pay a premium and you also accept a deductible, the first slice of any loss that you bear yourself.
- In return, there is a floor under what a disaster can cost you.
- A protective put is insurance on a holding.
- The investor owns the shares, then buys a put on them.
- A put gives the right to sell at the strike, so the strike is the floor.
- The premium is the price of the policy, and the gap between the purchase price and the strike is the deductible.
- Nora has worries about a results announcement, so Dev Rahman considers a put struck at forty five, costing one and a half.

16 How a protective put is built

Nora's protective put: 3 months, 15,000 premium

Own 10,000 shares at 50 → Buy 45 put for 1.50 → Wait for expiry

IN THE LESSON

- Building it takes two moves again, but this time the option is bought, not sold.
- Nora owns ten thousand shares bought at fifty.
- Dev Rahman buys ten thousand three month puts struck at forty five, paying one and a half each, a total of fifteen thousand.
- Now wait for expiry.
- If the share finishes above forty five, the put lapses unused, like an insurance policy with no claim, and she keeps all the upside.
- If it finishes below forty five, she exercises the put and sells at forty five, however far the share has fallen.
- So the objective is clear: keep the gains, and cap the loss.

17 Protective put: payoff and profit

Value at expiry = S + max(0, X − S); profit = value − S0 − p
Max loss = S0 − X + p; breakeven = S0 + p; gain unlimited

Value at expiry	Flat at X once S falls below X
Profit	Value, less cost, less premium
Maximum loss	Deductible plus the premium
Breakeven	Cost price plus the premium

IN THE LESSON

- The value of the combination at expiry is the share price plus whatever the put pays.
- The put pays the amount by which the price is below the strike, or nothing.
- So below the strike the value is flat at the strike, and above it the value is just the share price.
- Profit takes that value, subtracts the price paid for the shares, and subtracts the premium too, because this time the premium is spent.
- The most the investor can lose is the deductible plus the premium.
- There is no ceiling on the gain, because the shares can keep rising.
- And breakeven is the purchase price plus the premium, since the share must first recover the cost of the policy.

18 Example 6: the worst case and the breakeven

Cost 50; buy a 45 put for 1.50; 10,000 shares

Deductible	50 − 45	**5.00**
Premium paid	1.50 × 10,000	**15,000**
Maximum loss	5.00 + 1.50 = 6.50	**65,000**
Breakeven price	50 + 1.50	**51.50**

IN THE LESSON

- Put Nora's numbers in.
- The shares cost fifty, the strike is forty five and the premium is one and a half.
- The deductible is fifty minus forty five, five a share.
- Add the premium and the worst loss is six and a half a share, or sixty five thousand on ten thousand shares.
- No matter how far Tarn Foods falls, it cannot cost her more.
- The breakeven is fifty plus one and a half, fifty one and a half, because the share has to climb that far to repay the policy.
- And her total premium is one and a half times ten thousand, fifteen thousand, which is a certain cost whatever the share does.

19 The protective put on a graph

Chart: Profit per share vs Share price at expiry (30 to 70)

- Flat below 45: loss 6.50
- Breakeven 51.50
- Share alone

IN THE LESSON

- Here is the protective put as a picture.
- The axes are the same as before: the horizontal axis is the share price at expiry, the vertical axis is profit per share, and the dashed grey line is zero.
- The thin line is the shares alone, falling and rising one for one.
- The thick line is the protected position.
- Follow it from the right.
- It rises with the shares, but lower by the premium, because the premium is gone.
- That is why it crosses zero at fifty one and a half.
- Then at forty five it turns flat.
- Below the strike, every point lost on the shares is paid back by the put, so the loss stays at six and a half.

20 Example 7: choosing the strike

Limit 14% of 50 = 7.00 a share; three puts offered

48 put, premium 3.00	2.00 + 3.00	**5.00**
45 put, premium 1.50	5.00 + 1.50	**6.50**
40 put, premium 0.60	10.00 + 0.60	**10.60**
Her limit	14% × 50	**7.00**
Choice: the 45 put	15,000 cheaper than the 48	**Choose 45**

IN THE LESSON

- The key judgement is the strike.
- Nora can tolerate losing at most fourteen per cent of the fifty, which is seven a share.
- Dev Rahman prices three puts.
- The put at forty eight costs three and the worst loss is two plus three, five.
- The put at forty five costs one and a half and the worst loss is six and a half.
- The put at forty costs point six, but the worst loss is ten plus point six, ten point six, outside her limit.
- A lower strike is cheaper because the deductible is bigger.
- The forty five put is the cheapest that stays inside the limit.
- It costs fifteen thousand less than the forty eight put.

21 Example 8: the cost of insuring again

Renew the 45 put every quarter at 1.50 (illustration)

Premium for one quarter	1.50 × 10,000	**15,000**
Four quarters	4 × 15,000	**60,000**
Share of the 500,000 position	60,000 ÷ 500,000	**12%**
Per share, over the year	4 × 1.50	**6.00**

IN THE LESSON

- The real danger of a protective put is cost.
- A policy that is never used still has to be paid for.
- Suppose Nora renews the same protection every quarter at one and a half a share.
- That is four times one and a half, six a share over the year.
- On ten thousand shares, it is sixty thousand, and the position is worth five hundred thousand.
- So the insurance costs twelve per cent of the holding every year, and the shares have to earn that much just to break even.
- This is an illustration at a steady price, not a forecast.
- The lesson is to insure when there is a particular risk, like the results announcement, not as a habit.

22 Mistakes to avoid so far

- Dropping the bond: that leaves only a forward
- Forgetting income the options never receive
- Calling a covered call a hedge: it only cushions
- Forgetting the covered call caps the gain
- Cheapest put hides a big deductible; premiums drag

IN THE LESSON

- Let us collect the common mistakes from these three topics.
- One: leaving out the bond when copying a share from options, which leaves only a forward.
- Two: forgetting that a share paying income is worth more than the package, because the options never receive that income.
- Three: treating the covered call as a hedge, when the premium only cushions a fall and everything above the strike is given away.
- Four: forgetting the cap on the covered call when the share rallies.
- Five: choosing the cheapest put without looking at the deductible, or ignoring how repeated premiums drag on the return.

23 A story: how much of the ride is left?

- Delta: change in value for a small price move
- A share has a delta of one
- A short forward has a delta of minus one
- Share plus short forward: exposure of zero

IN THE LESSON

- Think of riding a bicycle on a windy day.
- How much the wind pushes you depends on how much of you is exposed.
- Delta measures that for a position.
- It is the change in value for a small change in the price of the underlying, and it tells you how many shares your position behaves like.
- A share has a delta of one.
- A short forward, a promise to sell at a fixed price, has a delta of minus one.
- Hold the share and sell the forward, and the two cancel, so the exposure is zero.
- Nora's son thinks a covered call does the same job.
- Dev Rahman will use delta to test that claim.

24 Deltas of the three positions

Share + short forward = 1 − 1 = 0; covered call = 1 − Δc
Protective put = 1 + Δp

Share delta	Always one
Call delta	Between zero and one
Put delta	Between minus one and zero
Position delta	Exposure per share held

IN THE LESSON

- Here are the three deltas.
- A share held long and hedged with a short forward has a delta of one minus one, zero.
- It is fully hedged, with neither upside nor downside.
- A covered call is a share and a written call.
- Writing the call takes away the call's delta, so the position is one minus the call delta.
- A call delta lies between zero and one, so the answer lies between zero and one too.
- A protective put is a share and a bought put, so we add the put delta, which is negative, between minus one and zero.
- Again the answer lies between zero and one.
- Both option positions sit between the bare share and the forward hedge.

25 How much exposure is left

Delta per share held

Share alone	**1.0**
Protective put	**0.7**
Covered call	**0.4**
Share + forward	**0**

Call delta 0.6; put delta −0.3

IN THE LESSON

- Now the picture.
- The vertical axis is delta per share held.
- The first bar is the share alone, with a delta of one.
- Dev Rahman takes a call with a delta of point six and a put with a delta of minus point three, as illustrations.
- The second bar is the protective put.
- One plus minus point three leaves point seven, so most of the exposure remains.
- The third bar is the covered call.
- One minus point six leaves point four, much less.
- The last bar is the share with a short forward, which is zero.
- So the protective put keeps the most exposure and the covered call keeps less.
- Neither goes all the way down to nothing.

26 Example 9: share-equivalents for Nora

10,000 shares; call delta 0.6 then 0.8; put delta −0.3 then −0.1

Short forward	10,000 × (1 − 1)	**0**
Covered call	10,000 × (1 − 0.6)	**4,000**
Protective put	10,000 × (1 − 0.3)	**7,000**
Covered call after rally	10,000 × (1 − 0.8)	**2,000**
Protective put after rally	10,000 × (1 − 0.1)	**9,000**

IN THE LESSON

- Turn those deltas into share equivalents for Nora's ten thousand shares.
- With a short forward, ten thousand times one minus one is nothing.
- With the covered call, ten thousand times one minus point six is four thousand, so she behaves as if she held only four thousand shares.
- With the protective put, ten thousand times one minus point three is seven thousand.
- Now the share rallies.
- The call, once less likely to fall worthless, has a delta of point eight, and the put, now less useful, has a delta of minus point one.
- The covered call falls to two thousand.
- The protective put climbs to nine thousand.
- The exposures have moved in opposite directions.

27 Two deltas that drift apart

Chart: Position delta vs Share price (today in the middle)

- Today 0.4, 0.7
- Rally 0.2, 0.9
- Call: falls
- Put: rises

IN THE LESSON

- Here is why those two numbers cannot be treated as fixed.
- The horizontal axis is the share price, with today's price in the middle.
- The vertical axis is the position delta per share.
- The line that falls is the covered call.
- As the price rises, the written call is more likely to be exercised, its delta climbs towards one, and the position delta drifts towards zero.
- The line that rises is the protective put.
- As the price rises, the put matters less, its delta shrinks towards zero, and the position delta climbs towards one.
- At today's price, the two lines read point four and point seven.
- After a rally they read point two and point nine.
- This is a sketch of the idea, not exact values.

28 Which tool for which client?

Matching the position to Nora's wish

If the client wants	Use
No exposure at all	Share plus short forward
Income, will sell at 55	Covered call
Upside kept, floor set	Protective put
Son: call equals forward	No: delta 0.4

IN THE LESSON

- Now Dev Rahman can answer the son.
- A share plus a short forward is the tool for a client who wants no exposure at all and accepts that she gives up both the gain and the loss.
- A covered call suits a client who wants income and who would sell at the strike, and who accepts a position that keeps part of the downside.
- A protective put suits a client who wants to keep the upside and sets a floor under the loss, and who pays for it.
- The son's claim fails because the covered call leaves a delta of point four, not zero.
- It cuts the exposure partly, and only a forward cuts it fully.

29 Recap: the lesson in five lines

- Call, short put and bond copy a share
- Covered call: income now, gain capped
- Protective put: floor set, premium paid
- Deltas: 1 − Δc, 1 + Δp, and 0 with a forward
- Only a forward removes the exposure completely

IN THE LESSON

- Let us pull the lesson together.
- First, a long call, a short put and a bond paying the strike copy a share that pays no income, and leaving out the bond leaves only a forward.
- Second, a covered call is a share plus a written call: it earns the premium, its profit is capped at the strike minus the cost plus the premium, and it breaks even below the cost price.
- Third, a protective put is a share plus a bought put: it sets a floor, its loss is capped at the deductible plus the premium, and its breakeven is above the cost price.
- Fourth, a covered call leaves a delta of one minus the call delta, a protective put one plus the put delta, and a hedge with a forward leaves nothing.

30 Try three quick questions

1. Share 62, call 5 (strike 63), rate 5%, 1 year. Put?
   Answer: PV(63) = 60. p = 5 + 60 − 62 = 3.00.
2. Buy at 40, write a 44 call for 1.60. Max profit, breakeven?
   Answer: 44 − 40 + 1.60 = 5.60; breakeven 40 − 1.60 = 38.40.
3. Protective put, 20,000 shares, put delta −0.25. Exposure?
   Answer: 20,000 × (1 − 0.25) = 15,000 share-equivalents.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A share is sixty two and pays no income.
- A one year call struck at sixty three costs five.
- The put has the same strike, and the rate is five per cent.
- What is the put?
- The strike's present value is sixty, so five plus sixty minus sixty two gives a put of three.
- Question two.
- An investor buys shares at forty and writes a call struck at forty four for one point six.
- What are the most she can make and the breakeven?
- Four plus one point six is five point six, and the breakeven is thirty eight point four.
- Question three.
- A protective put on twenty thousand shares has a put delta of minus point two five.
- How many share equivalents remain?
- Point seven five times twenty thousand is fifteen thousand shares.