CHARTA LEARNING · LESSON NOTES

Liabilities, ladders and alternatives

Portfolio Construction

1 A story: the school fees

- Judge the portfolio by the bills it must pay
- Start from the promises, then choose assets
- Size, timing and rate sensitivity of liabilities
- Not a race against a market index

IN THE LESSON

- Imagine a parent who knows that school fees will fall due every September for many years to come.
- A sensible parent does not ask which fund beat the market last year.
- The parent asks how to be sure the fees are paid.
- That is the idea behind liability driven investing, which we will call L D I.
- Most portfolios are judged against an index.
- A pension plan is judged against what it has promised to pay.
- So L D I builds the assets around the size, the timing and the interest rate sensitivity of those promises, and not around a market index.

2 Four kinds of liability

Sorted by what is known

Type	Amount, timing known?
Fixed bond payments	Both known
Life insurance benefit	Amount known only
Inflation-linked payment	Timing known only
Pay-linked pensions, claims	Neither known

IN THE LESSON

- Not every promise is equally certain, and the less certain it is, the harder it is to match.
- We can sort liabilities by whether the amount is known and whether the timing is known.
- A fixed schedule of payments, such as a bond the company has issued, has both known.
- A life insurance benefit has a known amount but an uncertain date.
- A payment linked to inflation or to a floating rate has a known date but an uncertain amount.
- Pension benefits that depend on future pay, and insurance claims from storms, are uncertain in both.
- Tomas Reyes starts by sorting Marlow's promises this way.

3 Example 1: Marlow's rate gap

Liabilities 400m, duration 12.5; bonds 150m, duration 8; assets 360m

Liability BPV	400m × 12.5 × 0.0001	**500,000**
Asset BPV	150m × 8 × 0.0001	**120,000**
Hedge ratio	120,000 ÷ 500,000	**24%**
Surplus today	360m − 400m	**−40m**
Yields fall 1 point	+12m − 50m	**−38m**

IN THE LESSON

- Here is Marlow.
- Its pension promises have a present value of four hundred million, with a duration of twelve and a half.
- The plan holds three hundred and sixty million of assets: one hundred and fifty million of bonds with duration eight, and the rest in shares, whose rate sensitivity the trustees treat as zero.
- Basis point value, or B P V, is the money change for one basis point.
- For the liabilities it is four hundred million times twelve and a half times point zero zero zero one, which is five hundred thousand.
- For the bonds it is one hundred and twenty thousand.
- The hedge ratio is one hundred and twenty over five hundred, or twenty four per cent.
- The plan is already forty million short.
- Now yields fall one point.
- Liabilities rise fifty million, assets only twelve, so the shortfall grows by thirty eight million.
- That is the gap Tomas Reyes must address.

4 Basis point value and the hedge ratio

BPV = PV × duration × 0.0001
Hedge ratio = asset BPV ÷ liability BPV

Present value	Value of the cash flows
Duration	Sensitivity to yield
Asset BPV	Money moved by a basis point
Liability BPV	What the promises move by
Hedge ratio	Share of rate risk covered

IN THE LESSON

- Now the two formulas behind that example.
- Basis point value is the present value, times the duration, times point zero zero zero one.
- It answers one question: if yields move by one basis point, how many dollars does the value change?
- Compute it for the liabilities and again for the assets.
- The hedge ratio is then asset B P V divided by liability B P V.
- A ratio of full cover means the surplus is protected against small, parallel moves in rates.
- Marlow's ratio is low, which means most of its rate risk is open.
- The ratio is a dial, not a switch.
- Trustees often choose a target short of full cover.

5 Closing the BPV gap

BPV, thousands per basis point

Liabilities	**500**
Assets today	**120**
With swaps	**350**

Target hedge ratio 70%

IN THE LESSON

- Here are three bars, in thousands of dollars per basis point.
- The vertical axis is B P V.
- The first bar is the liabilities: five hundred.
- The second is the plan's assets today: only one hundred and twenty.
- That wide gap is the unhedged rate risk.
- The third bar is the assets once Tomas Reyes adds a swap overlay: three hundred and fifty, which is seventy per cent of the liabilities.
- A swap, remember, is an agreement to exchange cash flows, and a receive fixed swap rises in value when yields fall, just as the liabilities do.
- Notice that nothing was sold to get there, so the shares stay in place to earn return.

6 Example 2: a swap overlay

Target hedge ratio 70%; swap BPV 1,000 per 1m notional

BPV needed	0.70 × 500,000	**350,000**
BPV to add	350,000 − 120,000	**230,000**
Swap notional	230,000 ÷ 1,000 per m	**230m**
Shares sold	overlay only	**none**
Yields fall 1 point	+35m − 50m	**−15m**

IN THE LESSON

- Now the proposal.
- The trustees want a seventy per cent hedge ratio but do not want to sell shares.
- Seventy per cent of five hundred thousand is three hundred and fifty thousand.
- The plan has one hundred and twenty thousand, so it must add two hundred and thirty thousand.
- A receive fixed swap carries about one thousand of B P V for each million of notional.
- So Tomas Reyes needs two hundred and thirty million of notional.
- Check the result.
- If yields fall one point, the assets now gain thirty five million, against fifty on the liabilities.
- The shortfall grows by fifteen million, instead of the much larger fall we saw before.
- The plan has not eliminated the risk, but it has cut it sharply.

7 The LDI process

Know the promises

Size of each bill → Timing of each bill → Rate sensitivity

Fund them

Long bonds → Cash flow match → Swap overlay

IN THE LESSON

- So the L D I process runs in two rows.
- The top row is about the promises.
- First, measure how large each liability is.
- Second, find when each is due.
- Third, find how sensitive its value is to interest rates.
- The bottom row is about the assets.
- One way to fund the promises is with bonds, bought long enough to match the sensitivity.
- Another is cash flow matching, which we study next, where each bond payment lines up with a bill.
- A third is an overlay of swaps or futures, which adds rate sensitivity without selling anything.
- Most plans mix these, and keep some return seeking assets for the growth the plan still needs.

8 The trap: matching duration alone

- Equal duration is only the first step
- Asset convexity should exceed liability convexity
- Curves can twist: yields do not move together
- Rate risk is cut, not removed

IN THE LESSON

- Here is the trap.
- Matching duration is the first step, not the last.
- Duration is a straight line approximation, and the liabilities curve away from that line.
- If the assets have less convexity than the liabilities, big moves in yields hurt the plan, whichever way yields go.
- So the assets should have convexity at least as large as the liabilities, which is why a portfolio of bonds spread around the liability dates tends to be safer than a single bond of the same duration.
- A second trap is the assumption that all yields move together.
- When the curve twists, a duration match can still open a gap.
- So rate risk is cut, not removed.

9 Tools for closing the gap

Four ways to hedge rate risk

Tool	Main point
Long bonds	Simple, uses cash
Cash flow matching	Bond pays each bill
Immunisation	Match whole-portfolio sensitivity
Swap or futures overlay	No sale, adds collateral needs

IN THE LESSON

- Let us put the common tools side by side so the choice is clear.
- Buying long bonds is simple, but it uses up cash that might have gone to return seeking assets.
- Cash flow matching ties each bill to a bond payment.
- Immunisation matches the sensitivity of the whole portfolio to the liabilities, so the plan can rebalance as time passes.
- Swap or futures overlays add rate sensitivity cheaply, without selling anything, but they bring collateral calls and counterparty exposure.
- Tomas Reyes recommends a mix for Marlow: some bonds, and a swap overlay to bring the hedge ratio up.

10 A story: the envelopes

- One envelope per bill, filled in advance
- Bond payments line up with the liabilities
- No view on interest rates is needed
- Marlow's pensioners: 2m, 2m, 3m in years 1 to 3

IN THE LESSON

- Think of a family that puts cash into labelled envelopes, one for each month's rent.
- Every bill has an envelope, and there is exactly enough in it.
- Nobody needs a view on prices.
- Cash flow matching is the same idea with bonds.
- Marlow also has a small group of pensioners whose benefits are fixed: two million at the end of year one, two million at the end of year two and three million at the end of year three.
- Tomas Reyes can buy bonds whose coupons and principal arrive on those very dates.
- Because each bill is met by a named cash flow, the plan does not depend on where interest rates go.

11 Building the match backwards

Work from the last bill

Cover the last bill →
Subtract its earlier coupons →
Step back and repeat

IN THE LESSON

- The trick is to build the portfolio backwards, from the last bill to the first.
- Start with the last liability.
- The longest bond is the only one that pays on that date, so buy exactly enough of it to cover the last bill.
- Next, notice that this long bond also pays coupons in the earlier years, so subtract them from those earlier bills.
- Then step back one year.
- What remains of that year's bill is covered by the bond maturing then.
- Repeat until the first year is done.
- Each step leaves a smaller amount for the shorter bonds.
- If you build forwards instead, you cannot know how much of the later coupons you will have.

12 How much of each bond to buy

Face = (liability − coupons from longer bonds) ÷ (1 + coupon rate)

Liability due	The bill due that year
Coupons from longer bonds	Already paid by longer bonds
Coupon rate	Annual coupon of that bond
Face to buy	Amount to buy at par

IN THE LESSON

- The calculation at each step is short.
- Take the liability due that year.
- Subtract the coupons that longer bonds already pay in that year.
- What remains must be met by the bond maturing that year, which pays its face value plus one coupon.
- So the face amount to buy equals what remains, divided by one plus the coupon rate.
- For the last year, there are no longer bonds, so the coupon deduction is zero.
- Notice that the answer is a face amount.
- If the bonds are priced at par, the cost equals the face.
- If they are not at par, the price per hundred of face must be applied as well.
- In all that follows we assume par.

13 Example 3: Marlow's three bills

Bills 2.000, 2.000, 3.000 (m); par bonds 3%, 4%, 5%

3-year bond	3.000 ÷ 1.05	**2.857**
2-year bond	(2.000 − 0.143) ÷ 1.04	**1.786**
1-year bond	(2.000 − 0.143 − 0.071) ÷ 1.03	**1.734**
Total cost	2.857 + 1.786 + 1.734	**6.377**
Coupons: 3-yr, 2-yr	5% × 2.857; 4% × 1.786	**0.143; 0.071**

IN THE LESSON

- Let us build it for Marlow, in millions.
- The bonds trade at par: one year at three per cent, two years at four, three years at five.
- Work backwards.
- The last bill is three.
- Divide by one point zero five and the three year face is two point eight five seven.
- It pays coupons of point one four three in each of years one and two.
- Year two's bill is two, less point one four three, which leaves one point eight five seven.
- Divide by one point zero four: the two year face is one point seven eight six, with a coupon of point zero seven one in year one.
- Year one needs two, less both coupons, which leaves one point seven eight six.
- Divide by one point zero three: one point seven three four.
- Total cost is six point three seven seven million.
- Every bill now has its own payment waiting.

14 What cash flow matching gives up

Where the match can break

Weakness	What goes wrong
Cost	Exact bonds are dear
Early cash	Reinvest at unknown rate
Call risk	Issuer repays early
Credit risk	Issuer cannot pay

IN THE LESSON

- A matched portfolio looks perfect, so it is worth asking where it falls short.
- First, cost.
- The exact bonds may be scarce or expensive, and the cost is the price of certainty.
- Second, timing.
- Real bills rarely fall on the dates of available bonds, so some cash arrives early and must be reinvested at an unknown rate.
- Third, call and prepayment risk.
- If an issuer repays early, the bond we counted on disappears.
- Fourth, credit risk.
- If an issuer defaults, a named bill goes unpaid.
- So the old claim that the portfolio is risk free needs care.
- It is free of interest rate risk, but not of credit and reinvestment risk.

15 Example 4: when the match breaks

2-year bond called at par after year 1; new rate 2%

Cash back at year 1	bond called at par	**1.786**
Needed in year 2	1.786 × 1.04	**1.857**
Rate for the year	2% instead of 4%	**−2 points**
Shortfall	1.786 × 0.02	**0.036**

IN THE LESSON

- Here is a call at work.
- Suppose the two year bond in Marlow's portfolio is callable, and the issuer repays it at par after one year.
- Marlow gets back one point seven eight six million, but must still meet a bill of one point eight five seven in year two, which is that amount times one point zero four.
- The bond would have paid four per cent for the year.
- Now the best available rate is two per cent.
- The shortfall is the amount returned times the gap in rates, two points, which comes to point zero three six million.
- It is small here, but it shows the lesson.
- Cash arriving early becomes a bet on future rates, and the match is no longer exact.

16 A story: the staircase

- Ten rungs: 1m in each maturity, 1 to 10 years
- Each year the shortest rung matures
- Reinvest the cash in a new ten-year bond
- Easy to explain, but no bill is matched

IN THE LESSON

- Picture a staircase with ten steps.
- A bond ladder spreads money roughly evenly across maturities, perhaps one to ten years.
- Marlow's retiree reserve puts one million into each maturity.
- Each year the shortest rung matures.
- The cash comes back, and Tomas Reyes buys a new ten year bond with it, so the ladder keeps its shape and the steps shift down by one.
- The structure is easy to explain to trustees, and it brings real benefits.
- But it solves a different problem from the one we met before.
- A ladder is not built to fund a particular bill.

17 How the ladder rolls

One year of the ladder

Equal rungs → Shortest rung matures → Reinvest at the long end

IN THE LESSON

- This is how the ladder works from year to year.
- In the first box, the money is split into equal rungs.
- Each rung is a bond with a different maturity.
- In the second box, one rung matures every year, so a regular slice of the portfolio turns into cash.
- In the third box, that cash is reinvested at the long end, in a bond of the longest maturity.
- After enough years, every rung is a bond that was bought at the long end, and the portfolio's return follows an average of rates across the whole rate cycle.
- This steady rolling is the heart of the ladder.

18 Duration, convexity and price change

$$\Delta P/P \approx -D \times \Delta y + \tfrac{1}{2} \times C \times (\Delta y)^2$$

$$D = T \div (1 + y);\ C = T(T + 1) \div (1 + y)^2$$

Modified duration	T ÷ (1 + y) for a zero
Convexity	Curvature of the price
Change in yield	Move in the yield
Price change	Estimated price change

IN THE LESSON

- To compare a ladder with a single bond, we need two measures.
- For a zero coupon bond maturing in T years, modified duration is T divided by one plus the yield.
- Convexity is T times T plus one, divided by one plus the yield, squared.
- A portfolio's duration and convexity are the weighted averages of its parts.
- Then the price change for a small move in yield is minus duration times the change, plus half of convexity times the change squared.
- The first term is the straight line.
- The second term is the curve, which is always helpful.
- More convexity means a smaller loss when yields rise and a bigger gain when they fall.

19 Example 5: ladder against bullet

10m ladder, rungs 1 to 10 years; flat 5% curve; bullet = one 5.5-year zero

Duration, both	5.5 ÷ 1.05	**5.24**
Convexity, ladder	avg of T(T+1) ÷ 1.05²	**39.9**
Convexity, bullet	5.5 × 6.5 ÷ 1.05²	**32.4**
Yields +1 point: ladder	−5.24% + 0.20%	**−5.04%**
Yields +1 point: bullet	−5.24% + 0.16%	**−5.08%**

IN THE LESSON

- Now Marlow's ten million.
- Use zero coupon bonds and a flat five per cent curve.
- The ladder holds rungs from one to ten years, so its average maturity is five and a half, and its modified duration is five and a half over one point zero five, which is five point two four.
- A single bond maturing at five and a half years has the same duration.
- But the ladder is more spread out, so its convexity is thirty nine point nine, against thirty two point four for that single bond.
- If yields rise one point, the ladder loses five point zero four per cent and the single bond five point zero eight.
- Better, but still a loss.
- The ladder also hands back cash each year.

20 A steep curve and the ladder

Chart: Yield (%) vs Years to maturity (1 to 10)

- Ten-year yield 5.0%
- Ladder average 3.8%
- One year 2.0%

IN THE LESSON

- Now return.
- The horizontal axis is years to maturity, one to ten.
- The vertical axis is yield.
- The curve is steep: a one year bond yields two per cent, a ten year bond five.
- The red flat line at the top is the ten year yield, five per cent.
- The grey flat line is the ladder's average yield across all ten rungs, three point eight per cent.
- So when the curve is steep, a ladder gives up yield compared with holding only long bonds.
- That gap is a cost of its balance, and it buys liquidity and spreading of rate risk.
- It also comes with a benefit over time.
- As short rungs mature and are replaced with ten year bonds, the average yield climbs.

21 Example 6: what the ladder costs and earns

Rung yields 2.0% rising to 5.0%; curve unchanged; 10m

Ladder average yield	sum of ten rungs ÷ 10	**3.8%**
Ten-year only	all in the long bond	**5.0%**
Yield given up	5.0% − 3.8%	**1.2 points**
Per year on 10m	1.2% × 10m	**120,000**
After first roll	(38.0 − 2.0 + 5.0) ÷ 10	**4.1%**

IN THE LESSON

- Let us put numbers on the yield.
- The ten rung yields rise from two per cent to five per cent.
- Their average is three point eight.
- A portfolio held entirely in ten year bonds would earn five.
- That gap is one point two points, which on ten million is one hundred and twenty thousand a year.
- Now roll.
- After the first year, the two per cent rung has matured and been replaced at five.
- The average rises to four point one.
- After ten rolls, if the curve does not change, every rung is a ten year yield, and the average is five.
- The cost is real, and it shrinks.

22 What a ladder offers and where it falls short

Strengths against limits

Feature	Verdict
Cash each year	Regular liquidity
Convexity	More than a bullet
Matches a liability?	No
Rate risk	Spread, not removed

IN THE LESSON

- Now we can weigh the ladder.
- On the plus side, it gives regular liquidity, because a rung matures each year.
- It spreads reinvestment across time, so no single date decides the outcome, and its return tends towards an average of rates over the cycle.
- It has more convexity than a single bond of equal duration.
- On the other side, it is not tied to any liability.
- Callable or amortising bonds can distort the rungs.
- In a steep curve it gives up yield compared with long bonds.
- And the biggest misunderstanding: it does not remove interest rate risk.
- It only spreads it, as we saw when yields rose.

23 Mistakes to avoid so far

- Matching duration alone leaves convexity gaps
- Cash flow matching still has credit and call risk
- A ladder spreads rate risk, it does not remove it
- A hedge ratio is a dial the trustees choose

IN THE LESSON

- Before the last topic, let us collect the traps.
- First, duration matching alone is not enough: convexity and non parallel moves can still open a gap.
- Second, cash flow matching is not risk free.
- It carries credit risk, call risk and reinvestment risk.
- Third, a ladder does not remove interest rate risk, and it does not fund any specific bill.
- It spreads the risk, it supplies cash each year, and in a steep curve it earns less than long bonds.
- Fourth, a hedge ratio is a dial.
- It reduces risk, and the trustees choose how far to turn it.
- Keep these in mind for the exam, and for the client.

24 A story: the toolbox

- Alternatives are many tools, not one
- Each is added for a different job
- Ask first what job the client needs done
- Marlow: pensions that rise with inflation

IN THE LESSON

- A toolbox holds a hammer, a saw and a spanner, and nobody would call them all the same tool.
- Alternative investments are like that.
- The label covers private equity, private credit, real estate, hedge funds, and real assets such as infrastructure, farmland, timberland and commodities.
- They are added for different jobs.
- So the question is never whether the portfolio should hold alternatives, but which job a particular client needs done.
- Tomas Reyes now asks that question for Marlow, whose pensions rise with inflation.

25 Four jobs for alternatives

The job decides the asset

Job	Usual alternatives
Capital growth	Private equity
Income	Private credit, core real estate
Diversification	Macro, relative-value funds
Inflation hedge	Infrastructure, land, commodities

IN THE LESSON

- Here are the four jobs.
- Capital growth is the job of private equity.
- It aims to beat listed shares, but remember that much of it is equity exposure magnified by debt, so it still moves with the stock market.
- Income is the job of private credit and core real estate, because their returns come mainly from interest and rent.
- Diversification, and a smaller fall in bad markets, is the job of hedge funds, especially macro and relative value styles.
- And an inflation hedge is the job of real assets, such as infrastructure, timberland, farmland and commodities, whose cash flows or prices tend to rise with inflation.

26 Matching the job to the client

If the client needs... then consider...

Client need	Alternative role
Pension needing income	Private credit, core real estate
Endowment seeking growth	Private equity
Inflation-linked liabilities	Infrastructure, real assets
Limit deep falls	Macro hedge funds

IN THE LESSON

- The right alternative depends on the client's objective, not on the label.
- A pension that needs income would look mainly at private credit and core real estate.
- An endowment that needs growth over a very long horizon might lean on private equity, accepting illiquidity.
- A plan with liabilities that rise with inflation would value infrastructure.
- And a client that cannot bear a deep fall would consider macro hedge funds.
- Marlow, with inflation-linked pensions and a need for steady cash, fits the income and inflation hedge rows.

27 Undoing the smoothing

$r = (R - \rho \times R_{-1}) \div (1 - \rho)$
σ = reported $\sigma \times \sqrt{((1 + \rho) \div (1 - \rho))}$

Reported return	From appraisals
Autocorrelation	Link to last period's value
Unsmoothed return	Closer to the real move
Unsmoothed volatility	Always above reported

IN THE LESSON

- There is one catch, and it matters for every alternative that is valued by appraisal.
- Appraisers lean on last period's value, so reported returns are smoothed.
- That makes volatility look too low and correlation with shares look too low.
- To undo it, take the reported return, subtract the autocorrelation times last period's reported return, and divide by one minus the autocorrelation.
- That gives the unsmoothed return.
- For volatility, multiply the reported figure by the square root of one plus the autocorrelation, over one minus it.
- The heavier the smoothing, the bigger the correction.
- It assumes true returns are unrelated from one period to the next.

28 Example 7: Marlow's real estate fund

Reported 2%, 3%, −1%; $\rho = 0.6$; reported volatility 5%

Year 2, unsmoothed	$(3 - 0.6 \times 2) \div 0.4$	**4.5%**
Year 3, unsmoothed	$(-1 - 0.6 \times 3) \div 0.4$	**−7.0%**
Volatility factor	$\sqrt{(1.6 \div 0.4)}$	**2.0**
Unsmoothed volatility	$5\% \times 2.0$	**10%**

IN THE LESSON

- Marlow is looking at a core real estate fund.
- Its appraisal based returns over three years were two, three and minus one per cent.
- The autocorrelation is estimated at point six.
- Year two unsmoothed is three, minus point six times two, divided by point four.
- That gives four and a half per cent.
- Year three is minus one, minus point six times three, divided by point four, so minus seven.
- The fund that reported a loss of one per cent actually lost nearer seven.
- The volatility factor is the square root of one point six over point four, which is two, so reported volatility of five per cent becomes ten.
- Smoothing hides real risk.

29 Diversification, reported against honest

Portfolio volatility (%)

All shares	**16.0**
Reported: 5, 0.25	**13.1**
Unsmoothed: 10, 0.6	**14.1**

80% shares, 20% real estate. Benefit 2.9, then 1.9 points

IN THE LESSON

- Does the fund still diversify?
- Take a portfolio of eighty per cent shares, with volatility of sixteen per cent, and twenty per cent real estate.
- On the vertical axis is portfolio volatility.
- The first bar is all shares: sixteen.
- The second uses the reported inputs, volatility five and correlation with shares point two five: the mix has volatility of thirteen point one, a big fall.
- The third uses honest inputs, volatility ten and correlation point six: fourteen point one.
- So the diversification benefit shrinks from two point nine points to one point nine.
- It is real, but smaller than the reported data promised.

30 Example 8: proposing and defending the plan

Assets 360m: bonds 150m, shares 210m

Private credit	5% × 360m	**18m**
Infrastructure	5% × 360m	**18m**
Total alternatives	18m + 18m	**36m = 10%**
Shares after	210m − 36m	**174m**
Private equity	plan needs cash, not growth	**none**

IN THE LESSON

- Now the proposal, and the defence.
- Tomas Reyes recommends ten per cent in alternatives: five per cent in private credit, for income, and five per cent in infrastructure, for the inflation link in the pensions.
- Five per cent of three hundred and sixty million is eighteen million each, thirty six million in all.
- The money comes from shares, which fall from two hundred and ten to one hundred and seventy four million.
- He does not recommend more private equity, because the plan needs steady cash and a tighter match, not extra equity risk.
- What could go wrong?
- The returns may be smoothed, so the diversification is smaller than reported, and the assets are hard to sell.
- So the weight stays modest.

31 Recap: the lesson in five lines

- LDI: hedge ratio = asset BPV ÷ liability BPV
- Cash flow match: build backwards, risks remain
- Ladder: spreads rate risk, does not remove it
- Alternatives: growth, income, diversify, hedge
- Unsmooth appraised returns before trusting them

IN THE LESSON

- Let us pull the lesson together.
- First, liability driven investing builds assets around the size, timing and rate sensitivity of the promises, and the hedge ratio is asset B P V over liability B P V.
- Second, cash flow matching builds a bond portfolio backwards from the last bill, so each bill has a named payment, with cost, reinvestment, call and credit risk remaining.
- Third, a ladder spreads maturities, gives regular cash and more convexity than a single bond, but does not match a bill or remove rate risk.
- Fourth, alternatives do four jobs: growth, income, diversification and inflation protection.
- Fifth, unsmooth their returns before trusting the diversification.

32 Try three quick questions

1. Liabilities 200m, duration 10; bonds 80m, duration 5. Hedge ratio?
   Answer: 40,000 ÷ 200,000 = 20%.
2. Last bill 5.25m; 5% par bond. Face to buy?
   Answer: 5.25 ÷ 1.05 = 5.0m.
3. Reported vol 3%, ρ = 0.6. Unsmoothed vol?
   Answer: 3 × √(1.6 ÷ 0.4) = 3 × 2 = 6%.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A plan has liabilities of two hundred million with duration ten, and eighty million of bonds with duration five.
- What is the hedge ratio?
- The liability B P V is two hundred thousand, the asset B P V is forty thousand, so the ratio is twenty per cent.
- Question two.
- The last liability is five point two five million, and the matching bond pays a five per cent coupon at par.
- What face amount do you buy?
- Divide five point two five by one point zero five, giving five million.
- Question three.
- A fund reports volatility of three per cent, with autocorrelation of point six.
- What is the unsmoothed volatility?
- The factor is two, so it is six per cent, which is a lot higher than reported, and that is the point.