CHARTA LEARNING · LESSON NOTES

Liability-driven bond portfolios

Portfolio Management

1 A story: the promise to pay

- Corrimore owes pensions for decades
- Assets must be there on every pay day
- One payment, or a whole schedule of them
- Rates move both sides of the balance sheet

IN THE LESSON

- Imagine you promise a friend a sum of money on a fixed date.
- You do not want to gamble, so you put the money somewhere safe that will grow to the right amount.
- Now imagine making that promise many times, each on a different date, for decades.
- That is the Corrimore Pension Scheme.
- Its adviser, Ines Cadmoor, must be sure the bonds the scheme owns can pay every pension when it falls due.
- The tricky part is that interest rates change both sides of the picture at once.
- They change what the bonds are worth, and they change what the promises are worth today.
- The job is to make the two move together.

2 One liability: immunisation

- Immunise: Macaulay duration = the horizon
- Rates up: price falls, reinvestment gains
- Rates down: price rises, reinvestment loses
- The two effects cancel on the pay date

IN THE LESSON

- Start with the simple case, a single payment.
- Suppose Corrimore must pay one lump sum on a known date.
- Interest rates can hurt in two ways.
- If rates rise, the bonds you hold fall in price, but the cash they pay out can be reinvested at the higher rate.
- If rates fall, the bonds rise in price, but reinvested cash earns less.
- Price risk and reinvestment risk pull in opposite directions.
- Immunisation sets them so that they cancel on the payment date.
- The rule is to choose bonds whose Macaulay duration equals the time left until the payment.
- Macaulay duration is the weighted average time until the bond's cash flows arrive.

3 Example 1: one payment, immunised

Pay 10m in 6 yrs at 4% yield: half in 3-yr, half in 9-yr zeros

Target at year six		**10.000m**
Duration of the pair	½ × 3 + ½ × 9	**6 years**
Rates to 5%, barbell		**10.004m**
Rates to 3%, barbell		**10.004m**
All in 9-year, rates to 5%		**9.717m**

IN THE LESSON

- Here is a small case.
- Corrimore owes ten million dollars in six years, and the going yield is four per cent.
- The adviser splits the money evenly between a zero coupon bond maturing in three years and one maturing in nine years.
- The average of three and nine is six, so the duration matches the horizon.
- Now rates jump to five per cent and stay there.
- The three year bond is reinvested at the higher rate, and the nine year bond is sold at year six at a lower price.
- Together they give ten point zero zero four million.
- If rates fall to three per cent the answer is the same, a hair above target.
- Now compare putting everything into the nine year bond.
- When rates rise it ends at only nine point seven one seven million, a clear shortfall.

4 Example 2: cash flow matching

Pay 10m at years 1, 2 and 3; bonds pay 5% a year, bought at par

Year-3 bond, face	10 ÷ 1.05	**9.524m**
Year-2 bond, face	(10 − 0.476) ÷ 1.05	**9.070m**
Year-1 bond, face	(10 − 0.476 − 0.454) ÷ 1.05	**8.638m**
Total cost today	sum of the three faces	**27.232m**
Check: present value at 5%	10 discounted three times	**27.232m**

IN THE LESSON

- Cash flow matching is the most literal method.
- You work backwards from the last payment.
- Corrimore owes ten million at the end of each of the next three years, and a bond pays five per cent a year at par.
- For year three, a bond with a face value of ten divided by one point zero five, nine point five two four million, pays exactly ten million with its last coupon.
- Its coupons cover part of what is owed earlier, so year two needs only nine point zero seven million, and year one eight point six three eight million.
- The total cost today is twenty seven point two three two million, equal to the present value of the three payments.
- Every payment is covered whatever rates do.
- The price is that bonds must exist with the right dates, and nothing is left to manage.

5 Four ways to fund a schedule

What each approach to a schedule matches

Approach	Matches
Cash flow matching	every payment
Duration matching	rate sensitivity
Horizon matching	both, by date
Contingent immunisation	a surplus floor

IN THE LESSON

- Corrimore rarely owes three tidy payments.
- It owes a schedule, a long run of different amounts on different dates.
- There are four ways to handle that.
- Cash flow matching buys bonds so that every payment is covered, as we just saw.
- Duration matching does not match each payment.
- It matches how the whole schedule reacts to interest rates.
- Horizon matching mixes the two: cash flow matching for the early years, where certainty is cheap, and duration matching for the distant years, where finding matching bonds is hard.
- Contingent immunisation lets the manager run an active strategy while there is spare surplus, and switches to immunisation if the surplus falls to a floor.
- Further down the table, there is more flexibility and less certainty.

6 Duration matching: three tests

BPV = PV × duration × 0.0001
Assets: PV ≥, BPV =, convexity ≥ liabilities

Present value	Assets cover the liabilities
Basis point value	Dollar change per basis point
Convexity	Higher for assets, not by much

IN THE LESSON

- Duration matching has three tests, and Corrimore must pass all of them.
- First, the assets must be worth at least as much as the liabilities today.
- Second, the money duration must match.
- Money duration is the basis point value, the dollar change in value when yields move one basis point.
- You find it by multiplying present value by duration by point zero zero zero one.
- Matching plain duration is not enough, because assets and liabilities can be different sizes.
- Third, the convexity of the assets should be higher than that of the liabilities, so big moves in either direction help rather than hurt.
- But not much higher, because extra convexity means cash flows spread out, and that brings back the risk of uneven moves along the curve.

7 Example 3: closing a gap with swaps

Liabilities 500m, duration 12; bonds 520m, duration 9; swap duration 8

Liability BPV	500m × 12 × 0.0001	**$600,000**
Asset BPV	520m × 9 × 0.0001	**$468,000**
Gap per basis point	600,000 − 468,000	**$132,000**
BPV per 100m of swap	100m × 8 × 0.0001	**$80,000**
Swap notional needed	132,000 ÷ 80,000 × 100m	**$165m**

IN THE LESSON

- Now Corrimore's numbers.
- The liabilities are worth five hundred million dollars with a duration of twelve.
- The bonds are worth five hundred and twenty million with a duration of nine.
- The assets already exceed the liabilities, so the first test is passed.
- The liabilities' basis point value is five hundred million times twelve times point zero zero zero one, which is six hundred thousand dollars.
- The bonds' is four hundred and sixty eight thousand.
- The gap is one hundred and thirty two thousand dollars, so the assets are too short.
- Rather than sell bonds, Ines uses receive fixed interest rate swaps.
- Each hundred million of notional adds eighty thousand of basis point value.
- She needs one hundred and thirty two over eighty, times a hundred million, which is one hundred and sixty five million of notional.
- Bond futures could do the same job.

8 Horizon matching, drawn

Split the schedule by date

Early years: match cash flows →
Later years: match duration →
Swaps close any gap

IN THE LESSON

- Horizon matching is the compromise.
- Look at Corrimore's schedule in two parts.
- The early payments are near and sure, and bonds can be found to cover them, so the adviser buys those with cash flow matching.
- That removes any worry about the next few years.
- The payments far in the future are different.
- There are few bonds that long, and the cost of matching each one is high.
- So for those, the adviser uses duration matching, with the three tests we just met.
- The result is certainty where it is cheap and flexibility where it is not.
- A swap overlay or bond futures can then close any remaining gap in basis point value, without selling the bonds.
- That keeps the portfolio intact while the hedge is tuned.

9 Example 4: contingent immunisation

Assets 520m; immunising needs 500m; active only while a cushion remains

Starting cushion	520 − 500	**$20m**
Active strategy loses		**$15m**
Cushion now	505 − 500	**$5m**
A further loss of 5m	500 − 500	**$0**
Action	the cushion is gone	**Immunise**

IN THE LESSON

- Contingent immunisation adds a safety net to active management.
- Corrimore's bonds are worth five hundred and twenty million dollars, and immunising the liabilities today would need only five hundred million.
- The difference, twenty million, is the cushion.
- While there is a cushion, the manager may run an active strategy, perhaps betting on a view about rates.
- Suppose the strategy loses fifteen million.
- The cushion shrinks to five million, so the manager carries on.
- Another loss of five million would use it all up, and at that moment the manager must switch to immunisation and lock in what remains.
- Notice that the floor moves, because the cost of immunising rises as rates fall, so the manager has to watch it every day.
- The trap is to treat the cushion as fixed.

10 From certainty to flexibility

- Cash flow matching
- Horizon matching
- Duration matching
- Contingent immunisation

IN THE LESSON

- Let us place the four methods on one line.
- At the left is cash flow matching, with the most certainty.
- Moving right, horizon matching gives up some certainty in the distant years and gains flexibility.
- Duration matching sits further along, because only the sensitivity to rates is matched, and the manager has room to choose bonds.
- At the far right is contingent immunisation, which allows a real active strategy until the floor is reached.
- As you move right, the portfolio is cheaper and more flexible, but it leans more on assumptions about how rates move.
- For a sponsor with little tolerance for shortfall, point left.
- For one with a big surplus and an appetite for active management, point right.

11 A story: the crash test

- Shock the curve: parallel, steeper, flatter, twisted
- Record the change in assets and in liabilities
- The difference is the change in the surplus
- Pick the hedge the sponsor can live with

IN THE LESSON

- Before a car goes on sale, engineers crash it at different speeds and from different angles.
- One test is never enough.
- Interest rates deserve the same treatment.
- Ines Cadmoor takes Corrimore's portfolio and shocks the yield curve in several ways.
- A parallel shift moves every maturity by the same amount.
- A steepening raises long rates more than short rates, and a flattening does the opposite.
- A twist moves the middle against the ends.
- For each shock she records two numbers: how much the assets change, and how much the liabilities change.
- The difference is the change in the surplus, and the surplus is what the sponsor cares about.
- A hedge is good if the surplus stays inside what the sponsor can tolerate in every test.

12 Sizing the hedge

Hedge needed = BPV gap ÷ BPV of one hedge unit
Hedge ratio = BPV assets ÷ BPV liabilities
Rates fall: surplus change = −(gap × bp)

BPV gap	Liabilities less assets
BPV of one hedge unit	For example, per $1m of swap
Hedge ratio	100% means fully hedged

IN THE LESSON

- Two ideas make the scenarios easy.
- First, the hedge you need.
- Take the gap in basis point value, liabilities less assets, and divide it by the basis point value of one unit of the hedge, such as one million of swap notional.
- That tells you how many units to hold.
- Second, the hedge ratio, which is the basis point value of the assets divided by that of the liabilities.
- At one hundred per cent the surplus is protected against small parallel moves.
- Below one hundred per cent, the scheme has chosen to keep a gap, and a gap is a bet that rates will rise.
- If rates fall, the liabilities grow faster than the assets, and the surplus falls by the gap times the number of basis points.
- If rates rise, the surplus gains.

13 Example 5: three choices for Corrimore

Larger section: liability BPV $1.2m, assets $0.6m; swap $900 per bp per $1m; budget $8m

Gap per basis point	1,200,000 − 600,000	**$600,000**
Notional for a full hedge	600,000 ÷ 900 × 1m	**$667m**
Stay at 50%, rates fall 50bp	−600,000 × 50	**−$30m**
Swaption, rates fall 50bp	−30 + 30 − 6	**−$6m**
Swaption, rates rise 50bp	+30 − 6	**+$24m**

IN THE LESSON

- Here are the choices for the scheme's larger section, a separate book from the one we just hedged.
- Its liabilities have a basis point value of one point two million dollars, and the bonds six hundred thousand, so the hedge ratio is fifty per cent.
- The gap is six hundred thousand a basis point.
- A swap adds nine hundred dollars per basis point for each million of notional, so a full hedge needs about six hundred and sixty seven million.
- If the scheme stays at fifty per cent and rates fall fifty basis points, the surplus falls thirty million.
- The third choice is a receiver swaption, an option to receive fixed, bought for a premium of six million.
- If rates fall, the swaption pays the gap, so the surplus falls only the six million premium.
- If rates rise, the swaption lapses, the partial hedge gains thirty million, and after the premium the gain is twenty four million.
- The hedging budget is eight million.

14 Three hedges, three shapes

Chart: Change in surplus ($m) vs Rate change (−50 to +50bp)

- Full swap hedge: 0
- Stay at 50%: −30 to +30
- Swaption: −6 to +24

IN THE LESSON

- The horizontal axis is the change in rates, from minus fifty basis points on the left to plus fifty on the right.
- The vertical axis is the change in the surplus in millions of dollars.
- The flat green line is the full swap hedge.
- The swaps gain what the liabilities gain, so the surplus stays put.
- The steep red line is staying at fifty per cent.
- It loses thirty million if rates fall fifty basis points, and gains thirty million if they rise by that much.
- That is a deliberate bet that rates will go up.
- The gold line, bending in the middle, is the swaption.
- After the premium it loses only six million on the left, but gains twenty four million on the right.
- The scheme pays for the shape.

15 Strategies side by side

What each strategy costs Corrimore

Strategy	Price of protection
Full swap hedge	no upside
Stay at 50%	$30m at risk
Receiver swaption	$6m premium
Collar	capped gain

IN THE LESSON

- Each choice suits a different sponsor.
- A full swap hedge locks in the funded status.
- It costs nothing up front, but it gives up any gain if rates rise, and the swaps may call for collateral.
- Staying at fifty per cent is a bet.
- It is free, but a fall in rates could cost thirty million.
- The receiver swaption costs a six million premium and keeps most of the gain if rates rise.
- A collar sells a second option to pay for part of the first.
- That lowers the cost, but it caps the gain, so it sits between the swaption and the full hedge.

16 Example 6: a steeper curve

BPV in $000 at 5/10/20 yrs: liabilities 100/200/300, assets 250/250/100

Parallel shift, either way	600 − 600 per bp	**$0**
Steepener: liabilities	+2.5 − 7.5	**−$5.0m**
Steepener: assets	+6.25 − 2.5	**+$3.75m**
Steepener: surplus	3.75 − (−5.0)	**+$8.75m**
Flattener: surplus	mirror image	**−$8.75m**

Steepener: 5-year −25bp, 10-year 0, 20-year +25bp

IN THE LESSON

- Corrimore's assets and liabilities each have a basis point value of six hundred thousand dollars, but they are spread differently along the curve.
- The liabilities carry more at twenty years, the assets more at five and ten.
- On any parallel shift, the surplus does not change.
- Now steepen the curve: the five year yield falls twenty five basis points, the ten year stays, and the twenty year rises twenty five.
- The liabilities gain two point five million at the short end and lose seven point five at the long end, a net fall in value of five million.
- The assets gain six point two five and lose two point five, a net gain of three point seven five.
- The surplus rises by eight point seven five million.
- A flattening does the reverse, and the surplus falls by that amount.

17 Choosing: four questions

- Does the cost fit the $8m hedging budget?
- Worst loss bearable? Staying at 50% risks $30m
- Keep the gain if rates rise, or lock it in?
- Repeat for steeper, flatter and twisted curves
- Corrimore's choice: the receiver swaption

IN THE LESSON

- How does Ines choose?
- She asks four questions.
- Is the cost inside the hedging budget of eight million dollars?
- A full swap hedge costs nothing up front, and the swaption premium is small enough, so both fit.
- Is the worst loss in the surplus one the sponsor can bear?
- Staying at fifty per cent could lose thirty million if rates fall, and Corrimore's sponsor says it cannot bear that.
- Does the sponsor want to keep the gain if rates rise?
- If yes, the swaption fits and the full hedge does not.
- And does the choice survive steepening, flattening and twisting?
- Corrimore wants protection and some upside, so Ines proposes the swaption.
- In a written answer, name the strategy, then defend it from the budget and the tolerance.

18 Duration is one risk, not all

- Duration covers a small, parallel shift
- It says nothing about everything else
- Assumptions, curve shape, spreads, plumbing
- Matching duration does not remove all risk

IN THE LESSON

- A smoke alarm detects smoke, not burglars.
- Duration matching is a smoke alarm.
- It tells Corrimore whether a small parallel shift in one yield curve will hurt, and for that job it works.
- It says nothing about the other ways a liability driven portfolio can go wrong.
- We have already met one: the steepening that moved the surplus by millions of dollars with the basis point values perfectly matched.
- In this section we meet four families of risk that survive a careful match.
- The trap is to believe that matching duration removes all risk.

19 Four risks that survive

The four families, and where each comes from

Risk that survives	Source
Model risk	the assumptions
Curve risk	uneven moves
Spread risk	credit spreads
Counterparty and liquidity	margin calls

IN THE LESSON

- Here are the four families.
- Model risk comes first.
- The liabilities are an estimate, resting on assumptions about the discount rate, pay growth, inflation and how long members will live.
- A hedge is only as accurate as those assumptions.
- Next is curve risk, also called structural risk.
- Assets and liabilities with the same duration can have cash flows in different years, so a steepening or a twist moves them by different amounts.
- Third is spread risk.
- Liabilities may be discounted on a corporate bond curve, while the hedge uses government bonds or swaps, so a change in credit spreads moves one and not the other.
- Last is the plumbing: swap counterparties that must be able to pay, margin calls that must be met in cash, and uncertainty about timing.

20 Example 7: model risk

Liabilities 500m, duration 12; bonds 520m; two assumptions revised

Discount rate down 0.25%	12 × 0.25% × 500m	**+$15m**
Longer lives: liabilities +2%	2% × 500m	**+$10m**
New liabilities	500 + 15 + 10	**$525m**
Surplus before	520 − 500	**$20m**
Surplus after	520 − 525	**−$5m**

IN THE LESSON

- Let us put numbers on model risk.
- Corrimore's liabilities are worth five hundred million dollars with a duration of twelve, and the bonds are worth five hundred and twenty million, so the surplus is twenty million.
- Now the scheme's actuary revises two assumptions.
- First, the discount rate is cut by a quarter of one per cent.
- Twelve times a quarter of one per cent times five hundred million is fifteen million of extra liability.
- Second, members are expected to live longer, which raises the liabilities by two per cent, another ten million.
- The liabilities are now five hundred and twenty five million.
- Markets have not moved at all, yet the surplus of twenty million has become a shortfall of five million.
- No duration hedge could have prevented it.

21 Example 8: spread risk and collateral

Liab 1,000m, duration 14, corporate curve; hedge: govt bonds + 500m swaps

Spreads narrow 40bp: liabilities	14 × 0.40% × 1,000m	**+$56m**
Government hedge	yields did not move	**$0**
Surplus	0 − 56	**−$56m**
Yields up 100bp: swap overlay	−10 × 1.00% × 500m	**−$50m**
Collateral due	cash or eligible assets	**$50m**

IN THE LESSON

- Now spread risk.
- Another scheme has liabilities of one thousand million dollars with a duration of fourteen, discounted on a corporate bond curve.
- They are hedged with government bonds and a five hundred million receive fixed swap overlay with a duration of ten.
- First, corporate spreads narrow by forty basis points while government yields stay put.
- The liabilities rise by fourteen times point four per cent times a thousand million, fifty six million.
- The government hedge does not change, so the surplus falls fifty six million with duration perfectly matched.
- Later, government yields and the corporate curve both rise one per cent.
- The swap overlay loses ten times one per cent times five hundred million, fifty million, and that must be posted as collateral at short notice.
- Neither problem was a duration mismatch.

22 A margin call, step by step

What happens to Corrimore

Yields rise 100bp → Swaps lose $50m → Collateral due in cash

If cash is short

Sell bonds quickly → Hedge shrinks → Gap reopens

IN THE LESSON

- Let us follow a margin call from start to finish.
- Yields rise one hundred basis points.
- The swap overlay loses fifty million dollars in market value.
- The swap counterparty asks for collateral, and the scheme must post it in cash or eligible securities, usually within a day or so.
- The matching gain on the liabilities is only on paper, so it brings in no cash.
- If Corrimore holds a liquid buffer, it pays and carries on.
- If it does not, it must sell bonds quickly, perhaps at poor prices.
- Selling bonds shrinks the hedge, and the gap reopens at the worst possible moment.
- There is a second danger: the counterparty itself may fail, which is why schemes spread swaps across several and hold a buffer.

23 Mistakes to avoid so far

- Matching duration alone and ignoring curve twists
- Matching plain duration, not money duration (BPV)
- Treating a swaption or a ratio under 100% as free
- Corporate-curve liabilities, government hedge: spread risk
- Forgetting collateral on the swaps

IN THE LESSON

- Let us collect the mistakes.
- One: matching duration on its own and ignoring curve twists, which is how a perfect match lost money in the steepener.
- Two: matching plain duration instead of the money duration, so a smaller asset pool is not hedged properly.
- Three: treating a swaption as free, or forgetting that a hedge ratio below one hundred per cent is a bet on rising rates.
- Four: hedging corporate curve liabilities with government bonds and ignoring spread risk.
- Five: forgetting collateral, so that a good hedge drains cash just when markets are stressed.
- Keep the five in mind and many exam traps disappear.

24 A story: copying a moving target

- An equity index fund can buy every share
- A bond index holds thousands of issues
- Many trade rarely, and the list keeps changing
- So we copy the risks, not the holdings

IN THE LESSON

- An equity index fund can usually buy every share in its index.
- A bond index fund almost never can.
- Think of a library whose shelves are restocked every month.
- Some books are borrowed so rarely that no one knows what they are worth today.
- Others vanish when they mature or are called, and new ones arrive as the issuers borrow again.
- A bond index is like that.
- It holds thousands of issues, many thinly traded, and its list changes constantly.
- So when Corrimore wants its bond portfolio to track an index, the realistic aim is not to copy the holdings.
- It is to copy the risks the index carries, and to know which risks those are.

25 Why a bond index is hard to copy

Four features that make tracking hard

Feature	Effect
Thousands of issues, many thin	can't hold all
Maturities, calls, new issues	constant churn
Weight = debt outstanding	big debtors lead
Prices often evaluated	stale quotes

IN THE LESSON

- Four features make bond indexes awkward.
- First, there are thousands of issues, and many are so thinly traded that buying one means paying a wide gap between the dealer's buying and selling prices.
- Second, the list never sits still, because bonds mature, are called, or are issued.
- Each change forces a trade.
- Third, the index weights each issuer by the debt it has outstanding.
- The most indebted issuers therefore carry the largest weights, which is odd, because borrowing more earns a bigger share.
- Fourth, many bonds have no recent trade, so the index uses evaluated prices, estimates built from similar bonds.
- Dealer liquidity is also uneven, so costs are high.

26 Weight follows debt outstanding

Index weight (%)

Issuer A	**40%**
Issuer B	**30%**
Issuer C	**20%**
Issuer D	**10%**

Debt outstanding: A 400, B 300, C 200, D 100 (example)

IN THE LESSON

- Here is the weighting rule in a picture.
- Four fictional issuers sit in a small index.
- Their debt outstanding is four hundred, three hundred, two hundred and one hundred million dollars, a round total.
- The vertical axis is the index weight in per cent.
- Issuer A has the most debt, so it gets forty per cent.
- Issuer B gets thirty, issuer C twenty and issuer D ten.
- Notice what this means.
- If issuer A borrows more next year, its weight rises, and anyone tracking the index must buy more of it.
- A tracker is pushed toward the biggest borrowers whether or not they are the best credits.
- That is one reason managers check issuer exposure as well as duration.

27 Return from key rate durations

Return ≈ − Σ (key rate duration × yield change)
Tracking difference = − Σ (KRD portfolio − KRD index) × yield change

Key rate duration	Sensitivity at one maturity
Yield change	Move at that same maturity
Tracking difference	Portfolio return less index

IN THE LESSON

- Total duration is one number, but a yield curve has many points.
- Key rate duration measures the sensitivity of a portfolio's value to a change in the yield at one maturity point, with the other points left alone.
- Add the key rate durations across the curve and you get the total duration.
- To estimate the return from any curve move, multiply each key rate duration by the yield change at that point, add them up, and put a minus sign in front.
- Now take two portfolios, the index and ours.
- Their tracking difference is the same sum, but using the gaps between their key rate durations.
- If every gap is zero, no curve move opens up a difference.
- If the gaps are big, a twist does.

28 Example 9: same duration, different return

Index 1.0/2.0/2.0/1.0 vs barbell 2.5/0.5/0.5/2.5 at 2/5/10/30 years

Duration: index = barbell	1+2+2+1 = 2.5+.5+.5+2.5	**6.0 each**
Index return	−(1.0×0.20 + 1.0×0.20)	**−0.40%**
Barbell return	−(2.5×0.20 + 2.5×0.20)	**−1.00%**
Tracking difference	−1.00 − (−0.40)	**−0.60%**

2-year and 30-year yields +20bp; 5 and 10 unchanged

IN THE LESSON

- Corrimore tracks an index whose key rate durations are one, two, two and one at two, five, ten and thirty years.
- To hold fewer bonds, the portfolio is a barbell, with two and a half, a half, a half and two and a half.
- Add them up and both have a duration of six, a perfect match.
- Now the two year and thirty year yields rise twenty basis points, while the five and ten year do not move.
- The index loses one times point two plus one times point two, which is point four per cent.
- The barbell loses two and a half times point two twice, one per cent.
- The portfolio trails by point six per cent, though the duration was matched.

29 Example 10: tracking error and spread

Barbell vs index; key rates independent, 0.50% a year each

Gap in each key rate	+1.5, −1.5, −1.5, +1.5	**±1.5**
Risk from each key rate	1.5 × 0.50%	**0.75%**
Tracking error, four rates	$\sqrt{(4 \times 0.75^2)}$	**1.50%**
Spread duration gap	5.0 − 4.0	**1.0**
Spreads widen 30bp	−1.0 × 0.30%	**−0.30%**

IN THE LESSON

- How big a risk is that mismatch in general?
- The barbell's gaps against the index, in each key rate, are plus one and a half, minus one and a half, minus one and a half and plus one and a half.
- Suppose, for simplicity, each key rate moves with a yearly standard deviation of half of one per cent, and the moves are unrelated.
- Each gap then contributes one and a half times half of one per cent, point seven five per cent.
- Squaring, adding the four and taking the root gives a tracking error of one and a half per cent.
- Real yields move together, so the true figure would differ.
- Now credit.
- If the portfolio has a spread duration of five and the index four, and spreads widen thirty basis points, the portfolio trails by a further point three per cent.

30 Ways to hold fewer bonds

Three ways to hold less than everything

Full replication → Stratified sampling → Optimisation

Risks all three try to match

Duration and key rates → Sector and quality → Issuer, cash flows, convexity

IN THE LESSON

- So how does a manager build the portfolio?
- There are three broad routes.
- Full replication buys every bond.
- It tracks closely, but it is rarely practical, because of thin trading and the cost.
- Stratified sampling sorts the index into cells, by sector, quality and maturity, then holds a few typical bonds in each cell so that the cell weights match the index.
- Optimisation uses a model to choose holdings that minimise expected tracking error, subject to costs and limits.
- All three try to match the same set of primary risk factors: duration and key rate durations, sector and quality weights, issuer exposure, the spread of cash flows over time, and convexity.
- The skill is deciding how much tracking error to accept for a given cost.

31 Tracking error by approach

Tracking error (% a year)

Replicate all	**0.05%**
Sample cells	**0.25%**
Duration only	**1.50%**

First two are illustrations; the third is Example 10

IN THE LESSON

- Here is the trade-off as bars.
- The vertical axis is tracking error in per cent a year.
- The first bar is full replication, with a tiny tracking error of point zero five per cent.
- That figure is an illustration, and the trading cost is the highest.
- The second bar is a stratified sample, with an illustrative tracking error of point two five per cent, at a much lower cost.
- The third bar is the barbell from Example ten, which matched total duration only.
- Its tracking error is one and a half per cent, six times the sample's.
- Matching one number is cheap, but it leaves the other risks open.
- The sensible choice is usually a sample or an optimiser that matches several risk factors.

32 Recap: the lesson in five lines

- Single payment: Macaulay duration = horizon
- Schedule: cash flow, duration, horizon, contingent
- Test parallel and twisted shocks; fit budget and tolerance
- Model, curve, spread and collateral risks remain
- Index: match key rates, spread, sector and quality

IN THE LESSON

- Let us pull the lesson together.
- First, one liability is immunised by matching Macaulay duration to the horizon, while a schedule can be funded by cash flow matching, duration matching, horizon matching or contingent immunisation, and swaps close gaps.
- Second, duration matching needs enough assets, equal basis point value and higher convexity.
- Third, test the hedge across parallel and non parallel shocks, and choose the strategy that fits the sponsor's budget and tolerance.
- Fourth, model, curve, spread, collateral and liquidity risks all survive a duration match.
- Fifth, bond indexes are hard to copy, so match the risk factors, not the holdings.

33 Try three quick questions

1. BPV gap $90,000; swap adds $60,000 per $100m notional. Notional?
   Answer: 90,000 ÷ 60,000 × 100m = $150m.
2. KRD at 10y: portfolio 3.0, index 2.0; 10y yield +10bp. Tracking difference?
   Answer: −(3.0 − 2.0) × 0.10% = −0.10%.
3. Liab 800m, duration 10, corporate curve, govt hedge. Spreads narrow 30bp. Surplus?
   Answer: Liabilities +10 × 0.30% × 800m = +24m; surplus −$24m.

IN THE LESSON

- Now try three quick questions yourself.
- Pause after each one.
- Question one.
- A scheme's basis point value gap is ninety thousand dollars, and a swap adds sixty thousand per hundred million of notional.
- How much notional is needed?
- Ninety over sixty times a hundred million is one hundred and fifty million.
- Question two.
- A portfolio has a key rate duration of three at ten years, the index has two, and the ten year yield rises ten basis points with nothing else moving.
- What is the tracking difference?
- Minus one times point one is minus point one per cent, so the portfolio trails by that much.
- Question three.
- Liabilities of eight hundred million with a duration of ten are discounted on a corporate curve and hedged with government bonds.
- Spreads narrow thirty basis points.
- The liabilities rise twenty four million and the hedge does not move, so the surplus falls with no change in government yields at all.