CHARTA LEARNING · LESSON NOTES

Passive bonds, benchmarks and forward rates

Portfolio Management

1 A story: four ways to follow the index

- Wintergill Mutual: 200m of new money for bonds
- Passive: follow the index, do not try to beat it
- Own all, own a sample, own a fund, or a contract
- Differ in cost, speed, customising, crisis behaviour

IN THE LESSON

- Picture Wintergill Mutual with two hundred million dollars of new money for bonds.
- Dana Ottoway is told to follow a bond index, a published list of bonds that stands for the market.
- Following an index is called passive, because nobody is trying to beat it.
- The only question is how to get the exposure.
- She can own all the bonds, own a carefully chosen sample, own a share in a fund that holds them, or hold a contract that pays what they pay.
- All four deliver the index.
- They differ in cost, in speed, in how far you can customise, and in how they behave when markets turn ugly.

2 Four routes to the index

Same exposure, four ways to get it

Route	Strength and weakness
Full replication	Exact; rarely feasible
Enhanced indexing	Cheaper; some tracking error
Funds and ETFs	Instant; price can leave NAV
Swaps and futures	Quick; counterparty, roll

IN THE LESSON

- Here are the four routes side by side.
- Full replication means buying every bond in the index.
- It is exact, but it is rarely feasible, because indexes are huge and many bonds trade rarely.
- Enhanced indexing holds a smaller set of bonds chosen to match the index's main risk factors, which cuts cost but leaves some tracking error, the gap between the portfolio's return and the index.
- Pooled vehicles, such as bond funds and exchange traded funds, give instant diversification and easy trading, though an exchange traded fund's price can drift away from the value of its holdings in a crisis.
- Derivatives, such as swaps and futures, give exposure quickly without owning bonds.

3 Example 1: why not buy every bond?

Index of 5,000 bonds; Wintergill has 200m

Bonds in the index		**5,000**
Smallest sensible lot		**1m**
Lots Wintergill can buy	200m ÷ 1m	**200**
Share of the index held	200 ÷ 5,000	**4%**
Equal slice per bond	200m ÷ 5,000	**$40,000**

IN THE LESSON

- Let us test full replication on Wintergill's money.
- The index holds five thousand bonds.
- Many bonds trade in blocks, and say the smallest sensible lot is one million dollars.
- Two hundred million buys only two hundred lots.
- That is four per cent of the index.
- Spread equally, the money would be forty thousand dollars a bond, far too small to trade at a fair price.
- So full replication is out.
- It is more practical for a small index of a few liquid government bonds, and for a very large investor.
- For Wintergill, with a broad index, Dana has to hold a sample.
- The next step is choosing that sample sensibly.

4 How enhanced indexing is built

From index to sample

Slice into cells → Pick bonds per cell → Match the risk factors → Watch tracking error

IN THE LESSON

- Enhanced indexing, sometimes called stratified sampling, works like a recipe.
- First, slice the index into cells by the things that drive bond returns: maturity, sector, credit rating and currency.
- Second, choose a few bonds for each cell, so that the sample has the same share of each cell as the index.
- Third, check that the sample matches the index on the main risk factors, above all its duration, how much its value moves when yields move, and its key rate durations, which show where along the curve that sensitivity sits.
- Fourth, monitor the tracking error and rebalance when it drifts.
- Fewer bonds mean lower trading and holding cost.

5 Example 2: matching key rates

150-bond sample; 10-year rate +1 point: sample trails 0.10%

2-year point	index 1.0 | sample 1.0	**gap 0.0**
5-year point	index 1.8 | sample 1.7	**gap −0.1**
10-year point	index 2.2 | sample 2.3	**gap +0.1**
30-year point	index 1.0 | sample 1.0	**gap 0.0**
Total duration	6.0 | 6.0	**gap 0.0**

IN THE LESSON

- Here is the check in numbers.
- The index has a total duration of six years, spread over four points on the curve: one year of duration at the two year point, one point eight at five years, two point two at ten years and one year at thirty years.
- Dana's sample of one hundred and fifty bonds has one, one point seven, two point three and one.
- Both total six point zero, so the overall duration matches.
- But the sample is a little short at five years and a little long at ten.
- If the ten year rate rises one point, the sample lags the index by one tenth of a point.
- That small gap is the price of holding fewer bonds.

6 Example 3: a fund in a stressed market

Bond ETF; Wintergill holds 20m at net asset value

Net asset value per share		**$100**
Stressed market price		**$97**
Discount to NAV	(100 − 97) ÷ 100	**3%**
Position		**$20m**
Cost of selling then	3% × 20m	**$0.6m**

IN THE LESSON

- Now the pooled route.
- Wintergill puts twenty million dollars into a bond exchange traded fund.
- Each share has a net asset value, the value of the bonds behind it, of one hundred dollars.
- On a calm day the market price sits right on that value.
- In a stressed market, sellers outnumber buyers and the price can slide below it.
- Say it trades at ninety seven.
- That is a discount of three per cent.
- If Wintergill must sell then, the discount costs it three per cent of twenty million, six hundred thousand dollars.
- The bonds were not worth less.
- The wrapper just got hard to sell at its fair value.

7 Example 4: a swap on the index

Cash 200m earns 3.0%; swap financing 3.0% + 0.20%

Cash income	3.0% × 200m	**$6.0m**
Swap financing paid	3.2% × 200m	**$6.4m**
Net drag against the index	6.0m − 6.4m	**−$0.4m**
Drag as a share of notional	0.4 ÷ 200	**0.20%**
Risks not priced here	counterparty, roll	**unpriced**

IN THE LESSON

- Last, the contract route.
- Suppose Wintergill already holds two hundred million in cash that earns three per cent, which is six million a year.
- It enters a total return swap on the index with that notional.
- It receives the index return and pays the financing rate, three per cent plus a dealer spread of point two.
- That is three point two per cent, or six point four million.
- Cash income less financing leaves a drag of point four million, which is point two per cent of the notional.
- Exposure arrives at once, with no bonds to buy.
- But the dealer must pay, so there is counterparty risk, and futures bring roll costs too.

8 Choosing the route

If the need is... lean toward...

If the need is	Lean toward
Small sum, trade at once	Pooled fund or ETF
Large, fit to liabilities	Enhanced indexing
Quick or temporary exposure	Futures or swap
Small, liquid index	Full replication

IN THE LESSON

- So which route does Dana pick?
- It depends on four things: the size of the portfolio, the cost, the need to customise, and the liquidity the client needs.
- A small amount that must be tradable at once points to a pooled fund.
- A large portfolio that must follow an index but also needs to fit liabilities points to enhanced indexing.
- Quick or temporary exposure, for example while a manager is being hired, points to futures or swaps.
- Full replication suits a small, liquid index.
- The trap is to think there is a single best route.
- Each one trades cost, accuracy and risk differently, and the client's needs decide.

9 A story: the yardstick

- A benchmark is the yardstick for a portfolio
- It also shapes how the portfolio is built
- The easy default: the broad aggregate index
- Dana must defend the choice for Wintergill

IN THE LESSON

- Imagine judging a runner against a yardstick.
- If the stick is the wrong length, the verdict is nonsense, however fast she ran.
- A benchmark is the yardstick for a portfolio.
- It sets the return the manager is measured against, and it shapes how the portfolio is built, because a manager who is judged against it will stay near it.
- For a bond mandate, the easy default is the broad aggregate index.
- But that index was built to describe the bond market, not to serve Wintergill Mutual.
- Dana must be able to say why a particular benchmark is right for this client.

10 What makes a benchmark sound

Seven tests, four groups

Group	Tests
Clear	Unambiguous and measurable
Practical	Investable and owned
Fitting	Suits style and current views
Timely	Specified in advance

IN THE LESSON

- A sound benchmark passes seven tests, which we can group in four.
- It is clear: unambiguous, so anyone can name its holdings and weights, and measurable, so its return can be calculated often.
- It is practical: investable, meaning the manager could actually hold it, and owned, meaning the manager accepts it and is accountable for results against it.
- It fits: appropriate to the manager's style, and reflecting the manager's current investment opinions, so the manager is not measured against bonds it would never buy.
- And it is timely: specified in advance, not chosen after the results are in.
- For a bond mandate there is one more test, which matters most: fit with the client.

11 Valuing a basis point

BPV = market value × duration × 0.0001

Market value	Dollars of bonds
Duration	Percent move per point
One basis point	One hundredth of a percent
Value of a basis point	Dollars per basis point

IN THE LESSON

- To compare a benchmark with a client's needs we need one number for interest rate risk in dollars.
- A basis point is one hundredth of one per cent.
- The value of a basis point, written B P V, is the market value of the bonds, times their duration, times point zero zero zero one.
- It tells you how many dollars the position gains or loses for a one basis point move in yields.
- Do the same sum for the liabilities and for the assets.
- If the two numbers are equal, a change in rates moves both by about the same amount and the surplus is protected, to a first order.

12 Example 5: matching Wintergill's liabilities

Liabilities 800m, duration 12; assets 800m

Liability BPV	800m × 12 × 0.0001	**$960,000**
Assets in aggregate (duration 6)	800m × 6 × 0.0001	**$480,000**
Assets in long index (duration 12)	800m × 12 × 0.0001	**$960,000**
Rates fall 1 point: surplus	−(960k − 480k) × 100	**−$48m**
Verdict	match the liabilities	**Long index**

IN THE LESSON

- Wintergill has eight hundred million dollars of liabilities with a duration of twelve, and eight hundred million of assets.
- Its liability value of a basis point is eight hundred million times twelve times point zero zero zero one, nine hundred and sixty thousand dollars.
- If the assets follow the aggregate index, with a duration of six, their figure is four hundred and eighty thousand.
- Now rates fall one point, which is one hundred basis points.
- Liabilities grow by nine hundred and sixty thousand a point, assets by only four hundred and eighty.
- The gap is four hundred and eighty thousand times one hundred, forty eight million dollars of lost surplus.
- A long index with duration twelve closes it.

13 Duration: need against index

Duration (years)

Liabilities	**12**
Aggregate index	**6**
Long index	**12**

Illustrative durations

IN THE LESSON

- Here are the durations as bars.
- The vertical axis is duration in years, and the first bar is what Wintergill actually has to fund.
- Its liabilities have a duration of twelve.
- The second bar is the broad aggregate index, with a duration of six.
- That figure reflects what governments and companies chose to issue, not what any investor needs, and it drifts as issuers lengthen or shorten their debt.
- So a benchmark that looked right last year can slip away from the client without anyone deciding anything.
- The third bar is a long government and credit index, also twelve.
- Its credit quality is high, which suits an insurer with little room for loss.
- It resembles the liabilities, and that is the point.

14 Defending the choice

Valid does not mean suitable

Candidate benchmark	Verdict for Wintergill
Aggregate: duration 6	Valid, poor fit
Short index: duration 3	Valid, poor fit
Long gov-credit: duration 12	Valid, good fit

IN THE LESSON

- Now defend the choice.
- We put each candidate against the client's two needs: a horizon, set by the liabilities, and a tolerance for loss.
- All three candidates are valid in the sense of the tests, because each is published, measurable and investable.
- Validity is necessary but not enough.
- The aggregate index, with a duration of six, is valid but too short.
- A short index, with a duration of three, is valid but shorter still.
- The long government and credit index, with a duration of twelve and high credit quality, matches the liabilities and the insurer's caution.
- Dana writes it down: a benchmark chosen because it resembles what the client owes.

15 Example 6: the wrong yardstick

Portfolio duration 12; yields move 1 point

Active duration vs aggregate	12 − 6	**6 years**
Return gap, 1-point move	6 × 1%	**6.0%**
Active duration vs long index	12 − 12	**0 years**
Return gap, same move	0 × 1%	**0.0%**
Verdict	judge against the fit	**Long index**

IN THE LESSON

- There is a second reason the choice matters.
- Suppose Dana builds a portfolio with duration twelve to match the liabilities.
- Judge it against the aggregate index, and her active duration is twelve minus six, six years.
- If yields move one point, the portfolio and the index differ by six per cent.
- She looks reckless, though she did exactly what the client needed.
- Judge her against the long index and active duration is zero, and so is the gap.
- The benchmark decides whether good work looks like risk.
- That is why it must be agreed in advance, owned by the manager, and matched to the client.

16 Mistakes to avoid so far

- Assuming a fund always trades at net asset value
- Forgetting swap counterparty risk, financing and roll
- Picking the aggregate index out of habit
- Believing a valid benchmark is a suitable one
- Judging a manager against a benchmark that misfits

IN THE LESSON

- Let us collect the common mistakes.
- One: assuming a bond fund always trades at the value of its holdings, when in a stress it can sit at a discount.
- Two: forgetting that swaps and futures bring counterparty risk, financing cost and roll.
- Three: choosing the broad aggregate index out of habit, without asking whether its duration and credit mix suit the client.
- Four: believing that a valid benchmark is automatically a suitable one.
- Five: judging a manager against a benchmark that does not match the client, so that sound work looks like a bet.

17 A story: the bond's year

- Return = income + price change
- Yields up, prices down
- Duration is the first guide, for small moves
- Big moves, twists and options need more

IN THE LESSON

- Think of a bond portfolio as a rented flat.
- Over a year you collect rent, and you also find the flat has gone up or down in value.
- A bond portfolio's return works the same way.
- There is income, the coupons and the gain from the bond ageing.
- And there is price change, which comes from benchmark yields moving.
- Yields and prices move in opposite directions.
- Duration tells us roughly how much, for small moves.
- But when yields move a long way, when the curve twists instead of shifting, or when the bonds have options inside them, duration alone gives a wrong answer.
- Dana needs the full picture.

18 Duration and convexity

$$\%\Delta P \approx -D \times \Delta y + \tfrac{1}{2} \times C \times (\Delta y)^2$$

Price change	Percent change in price
Modified duration	First order sensitivity
Yield change	Change in benchmark yield
Convexity	Second order, bends the line

IN THE LESSON

- Here is the approximation.
- The percentage change in price is minus modified duration times the change in yield, plus one half times convexity times the change in yield squared.
- Duration is the first order term: yields rise, the price falls.
- Convexity is the second order term.
- For a bond with no options it is positive, so it adds to the result whether yields rise or fall.
- It makes the loss from a rise smaller and the gain from a fall larger.
- Because it uses the square of the move, it matters little for small moves and a great deal for large ones.
- That is why larger moves favour more convex portfolios.

19 Example 7: a one point move

Duration 7; convexity +60 or −60; yields move 1 point

Convexity term	½ × 60 × 0.01²	**0.30%**
Option-free, yields +1pt	−7.00 + 0.30	**−6.70%**
Option-free, yields −1pt	+7.00 + 0.30	**+7.30%**
Mortgage portfolio, yields +1pt	−7.00 − 0.30	**−7.30%**
Mortgage portfolio, yields −1pt	+7.00 − 0.30	**+6.70%**

IN THE LESSON

- Dana's portfolio has a modified duration of seven and a convexity of sixty.
- A colleague runs a mortgage portfolio with the same duration but a convexity of minus sixty, because the borrowers can repay early.
- Yields move one point.
- The convexity term is a half times sixty times point zero one squared, which is point three per cent.
- If yields rise, Dana's portfolio loses seven but gains point three back, minus six point seven.
- If they fall, it gains seven point three.
- The mortgage portfolio is the mirror image.
- It loses seven point three when yields rise and gains only six point seven when they fall.
- Negative convexity hurts both ways.

20 Duration is a straight line

Chart: Change in price vs Change in yield (points, −3 to +3)

- Positive convexity: above the line
- Duration alone
- Negative convexity: below it

IN THE LESSON

- Here is the picture.
- The horizontal axis is the change in yield, from minus three points on the left to plus three on the right.
- The vertical axis is the change in price.
- The grey straight line is duration alone: every point of yield rise costs the same amount.
- The green curve is the portfolio with positive convexity.
- It sits above the line on both sides, so it loses less when yields rise and gains more when they fall.
- The red curve is the callable or mortgage portfolio, below the line on both sides.
- When yields fall, borrowers refinance and the bonds are repaid just as they are most valuable.
- When yields rise, repayments slow and the portfolio lengthens just as the losses arrive.

21 Example 8: a three point move

Duration 7; convexity ±60; yields move 3 points; gap 5.4

Convexity term	½ × 60 × 0.03²	**2.70%**
Option-free, yields +3pts	−21.0 + 2.7	**−18.3%**
Option-free, yields −3pts	+21.0 + 2.7	**+23.7%**
Mortgage portfolio, yields +3pts	−21.0 − 2.7	**−23.7%**
Mortgage portfolio, yields −3pts	+21.0 − 2.7	**+18.3%**

IN THE LESSON

- Convexity grows with the square of the move, so let us try three points.
- The convexity term is a half times sixty times point zero three squared, which is two point seven per cent.
- If yields rise three points, duration says minus twenty one.
- Dana's portfolio loses eighteen point three after the convexity help.
- The mortgage portfolio loses twenty three point seven.
- If yields fall three points, Dana gains twenty three point seven and the mortgage portfolio only eighteen point three.
- The gap between the two portfolios is five point four points of return, from a pair that looked identical on duration.
- That is the exam trap: relying on duration alone when the move is large.

22 How the curve can move

Three ways the curve moves

Curve change	What happens
Level	Whole curve shifts in parallel
Slope	Curve steepens or flattens
Curvature	Middle moves against the ends

IN THE LESSON

- Yields on different maturities do not always move together, so curve changes are described in three parts.
- Level is a parallel shift, with every yield up or down by the same amount.
- It is the biggest mover, and it is what duration measures.
- Slope is the curve steepening or flattening, with the short and long ends drifting apart or together.
- Curvature, also called shape, is a butterfly move, where the middle of the curve moves against the two ends.
- A portfolio's return depends on where along the curve its exposure sits, so two portfolios with the same duration can react very differently to a slope or curvature move.

23 Example 9: a steepening curve

Duration 7.0 across four points; curve steepens

2-year: 1.2; yield −0.50	−1.2 × (−0.50)	**+0.60%**
5-year: 2.0; yield −0.25	−2.0 × (−0.25)	**+0.50%**
10-year: 2.6; yield 0.00	−2.6 × 0.00	**0.00%**
30-year: 1.2; yield +0.50	−1.2 × 0.50	**−0.60%**
Total price change		**+0.50%**

IN THE LESSON

- Now see why spreading duration matters.
- Dana's portfolio has a duration of seven, split across four points: one point two at two years, two at five years, two point six at ten years and one point two at thirty.
- The curve steepens.
- The two year yield falls half a point, the five year falls a quarter, the ten year does not move and the thirty year rises half a point.
- For each point, the price change is minus the key rate duration times the yield change.
- That is plus point six, plus point five, nothing, and minus point six.
- The total is plus point five per cent.
- A single duration number could never have told her that.

24 Example 10: rolling yield

Coupon 4.0; rolldown 0.3; duration 7; convexity ±60

Rolling yield	4.0 + 0.3	**4.3%**
Yields unchanged	4.3 + 0	**+4.3%**
Yields +1pt, option-free	4.3 − 7.0 + 0.3	**−2.4%**
Yields −1pt, option-free	4.3 + 7.0 + 0.3	**+11.6%**
Yields +1pt, mortgage	4.3 − 7.0 − 0.3	**−3.0%**

IN THE LESSON

- Return is not only price change.
- Over a year, Dana's portfolio earns its rolling yield: the coupon income plus rolldown, the gain as bonds age along an upward sloping curve and are valued at lower yields.
- This is often called carry, counted here before any financing cost.
- Her coupon income is four per cent and rolldown is point three, so rolling yield is four point three per cent.
- If yields do not move, that is the return.
- If yields rise one point, add the price effect, minus seven for duration and plus point three for convexity, and the year returns minus two point four.
- If they fall one point, plus eleven point six.
- The mortgage portfolio, with yields up one point, returns minus three.

25 A story: agreeing with the weather forecast

- Manager's view = what the curve already implies
- No bet on level, slope or shape
- Instead, earn the curve's static return
- Each strategy has a catch

IN THE LESSON

- Suppose the weather forecast says rain, and you agree.
- You do not bet against it.
- You just take an umbrella and carry on with your day.
- Now suppose Dana's view of interest rates is the same as what the market curve already implies.
- She has no view to bet on, so she does not take a position on the level, the slope or the shape of the curve.
- That does not mean doing nothing.
- An upward sloping curve pays you for holding longer bonds and letting them age.
- The strategies that collect that pay earn what we call the curve's static return, and each has a catch.

26 Strategies that earn the static return

All rely on the curve staying put

Strategy	How it earns
Buy and hold	Coupons at target duration
Roll down	Yield falls as bond ages
Repo carry, swaps	Borrow short, hold longer
Covered calls	Option premium collected

IN THE LESSON

- Here are the usual strategies.
- Buy and hold at a target duration collects the coupons, with the duration set by the client's needs.
- Rolling down holds bonds whose yields fall as they age along an upward sloping curve, so their prices rise.
- A repo financed carry trade borrows short at the low repo rate to hold a higher yielding longer bond.
- Long futures, or receiving the fixed rate on a swap, are leveraged ways of doing much the same thing.
- And selling options, such as covered calls, collects premium in return for capping the upside.
- All of them depend on one thing: the curve staying roughly where it is.

27 What the curve implies: forward rates

f = (1 + z2)² ÷ (1 + z1) − 1

One-year rate	Rate for year one, today
Two-year rate	Annual rate for two years
Forward rate	Implied rate for year two

IN THE LESSON

- To see what the curve is predicting we need forward rates.
- A forward rate is the rate for a future period that is implied by today's rates.
- Take the one year rate and the two year rate.
- Investing for two years should give the same result as investing for one year and then rolling into the one year rate that applies a year from now.
- So one plus the two year rate, squared, equals one plus the one year rate, times one plus the forward rate.
- Solve for the forward rate: one plus the two year rate, squared, divided by one plus the one year rate, minus one.
- This is the rate the market is pricing for the second year.

28 Today's curve and the forward curve

Chart: Yield (%) vs Maturity (years)

- Today's curve
- Implied for a year ahead
- Two-year: 4.0%
- One-year today: 3.0%
- Forward one-year: 5.01%

IN THE LESSON

- Here are the two curves.
- The horizontal axis is maturity in years, and the vertical axis is the yield.
- The solid green line is today's curve.
- It starts at three per cent at one year and climbs, more slowly as maturity grows.
- A two year bond yields four per cent.
- Now the red line, which is the curve that forward rates imply for one year from now.
- It sits well above today's curve at the short end: the one year rate a year from now is five point oh one per cent.
- So the forwards are not a forecast of a calm, unchanged world.
- They say short rates will rise, by just enough to cancel the gain from rolling down.

29 Example 11: rolling down, and the catch

1-year rate 3.0%; 2-year rate 4.0%; zero-coupon bond

Price today	100 ÷ 1.04²	**92.456**
Forward rate for year two	1.04² ÷ 1.03 − 1	**5.01%**
Return if the curve stays put	(100 ÷ 1.03) ÷ 92.456 − 1	**5.01%**
Sale price if forwards come true	100 ÷ 1.0501	**95.229**
Return if forwards come true	95.229 ÷ 92.456 − 1	**3.00%**

IN THE LESSON

- Test rolldown with numbers.
- The one year rate is three per cent and the two year rate is four.
- Dana buys a two year zero coupon bond for ninety two point four five six, and plans to sell it in a year.
- If the curve stays put, it is then a one year bond, worth one hundred divided by one point oh three, so the return is five point oh one per cent, against three on a one year bond.
- That is rolldown.
- But the forward rate for year two is also five point oh one.
- If the forwards come true, the bond sells for ninety five point two two nine, and earns exactly three per cent.
- The extra return vanishes.

30 Two outcomes for the same bond

Return over the year (%)

2-year, curve stays	**5.01%**
2-year, forwards	**3.00%**
1-year bond	**3.00%**

Same bond, two futures

IN THE LESSON

- Compare the outcomes.
- The vertical axis is the return over the year, in per cent.
- The first bar is the two year bond if the curve stays put: five point oh one.
- The second is the same bond if the forwards come true: three point oh oh.
- The third is the one year bond, which earns three whichever way the curve goes, because it simply matures.
- So the bond's extra return over the one year bond is not free.
- It is the reward for the curve being more stable than the forwards say.
- A manager who agrees with the market should size the trade knowing that, since the forwards describe a world where rolldown and carry earn nothing extra.

31 Example 12: a repo carry trade

Equity 20m; buy 100m of 2-year bonds at 4.0%; repo 3.0%

Bond income	4.0% × 100m	**$4.0m**
Repo cost on 80m borrowed	3.0% × 80m	**$2.4m**
Carry on Wintergill's equity	(4.0 − 2.4) ÷ 20	**8.0%**
Yields +1pt: price loss	1.9 × 1% × 100m	**– $1.9m**
Result on equity	(1.6 − 1.9) ÷ 20	**−1.5%**

IN THE LESSON

- Finally, the leveraged version.
- Wintergill gives Dana twenty million of its own money.
- She borrows eighty million at the repo rate of three per cent and buys one hundred million of two year bonds yielding four per cent.
- Bond income is four million.
- The repo cost is three per cent of eighty million, two point four.
- The carry is one point six million on twenty million of equity, eight per cent, double the four per cent unlevered.
- But the trade works only while the curve holds.
- The bonds have a duration of one point nine.
- If yields rise one point, the price falls one point nine million, and the year returns minus one point five per cent.

32 Recap: the lesson in five lines

- Four routes: replicate, sample, fund, contract
- Benchmark: valid tests plus fit to the liabilities
- Return: rolling yield, duration, convexity, curve moves
- No view: roll down, carry, sell options
- Forwards realised: the extra return disappears

IN THE LESSON

- Let us pull the lesson together.
- First, passive bond exposure comes by full replication, enhanced indexing, pooled funds or derivatives, and the right one depends on size, cost, customisation and liquidity.
- Second, a sound benchmark passes the validity tests and, for a bond mandate, resembles the client's liabilities in duration and credit quality.
- Third, a bond portfolio's return is rolling yield plus a price change of minus duration times the yield change, plus half convexity times the yield change squared, and curve moves matter too.
- Fourth, with no view against the market, earn the static return.
- Fifth, forward rates remind us that rolldown and carry are not free.

33 Try three quick questions

1. Duration 8, convexity 50; yields +2 points. Price change?
   Answer: −16 + ½ × 50 × 0.02² × 100 = −16 + 1 = −15%.
2. 1-year rate 2%, 2-year rate 3%. Forward rate for year two?
   Answer: 1.03² ÷ 1.02 − 1 = 4.01%.
3. Liab 500m, duration 10; assets 500m, duration 5. BPV gap?
   Answer: $500,000 − $250,000 = $250,000 per basis point.

IN THE LESSON

- Now try three quick questions.
- Pause after each one.
- Question one.
- A portfolio has a modified duration of eight and a convexity of fifty.
- Yields rise two points.
- What is the price change?
- Minus eight times two is minus sixteen.
- The convexity term is a half times fifty times point zero two squared, which is one per cent.
- So the change is minus fifteen per cent.
- Question two.
- The one year rate is two per cent and the two year rate is three.
- What is the forward rate for year two?
- One point zero three squared over one point zero two, minus one, is four point oh one per cent.
- Question three.
- An insurer has five hundred million of liabilities with a duration of ten, and five hundred million of assets with a duration of five.
- What is the shortfall in the value of a basis point?
- Liabilities give five hundred thousand dollars, the assets two hundred and fifty thousand, so the shortfall is two hundred and fifty thousand.
- It is a poor match.