CHARTA LEARNING · LESSON NOTES

Systematic risk, beta, the CAPM and the security market line

CFA Level I · Portfolio Management

You will learn: How to separate systematic from nonsystematic risk, calculate beta, use the capital asset pricing model (CAPM) to find a required return, read the security market line and alpha, and know where the model falls short.

Running example: Examples use made-up shares: Tern Brewing (risk 30%, beta 1.2), Linden Utilities (risk 20%), Quill Mining (risk 30%) and Orchid Tech (risk 40%). The market's standard deviation is 15%, the risk-free rate 3% and the expected market return 8%.

1. Systematic and nonsystematic risk

- Systematic (market) risk comes from forces hitting the whole market: interest rates, growth, inflation, policy. It cannot be diversified away. Nonsystematic (company specific, diversifiable) risk comes from one company or industry, such as a strike or a fire, and diversifies away at almost no cost.
- As shares are added, portfolio risk falls quickly then more slowly, heading towards a floor set by systematic risk and never below it.
- A share's variance splits into a market part and a company-only part. Because the parts are unrelated, their variances add.
- Only the systematic part is paid for: the market will not pay a premium for a risk anyone can remove for free. Total volatility is not the price tag.

Variance of a share = $\beta^2 \times \sigma m^2 + \sigma\varepsilon^2$ (systematic + nonsystematic)

SHARES WITH RISK 30% AND CORRELATION 0.3 (VARIANCE 900 ÷ N + (1 − 1/N) × 0.3 × 900), THEN TERN'S SPLIT

1 / 2 / 10 shares	variance 900 / 585 / 333	**30.0% / 24.2% / 18.2%**
40 shares / very many	variance 285.75 / 0.3 × 900 = 270	**16.9% / 16.4%**
Tern total variance	30^2	**900**
Systematic part	$1.2^2 \times 15^2 = 1.44 \times 225$	**324 (36%)**
Nonsystematic part	900 − 324	**576 (64%), SD 24.0%**

2. Beta

- Beta says how much a share's return tends to move for each one point the market's return moves (the market is stood in for by a broad index). Beta 1 moves one for one, above 1 amplifies, below 1 dampens.
- With beta 1.2 and the market up 10%, the share tends to rise about 12%. Beta 0 means no link; a negative beta (rare) moves opposite.
- Beta is the slope of the line relating share returns to market returns. Volatility says how much a share swings; beta says how much of that swing belongs to the market. Same volatility can mean very different betas.
- Beta is an estimate from past data. It depends on the index, the period and the frequency of returns, so treat it as a rough guide.

Beta = Cov(share, market) ÷ Var(market) = ρ × σshare ÷ σmarket

BETAS (MARKET VARIANCE $15^2 = 225$)

Tern, correlation 0.6	covariance 0.6 × 30 × 15 = 270; 270 ÷ 225	**1.2**
Linden, correlation 0.6	covariance 0.6 × 20 × 15 = 180; 180 ÷ 225	**0.8**
Quill, risk 30%, correlation 0.1	0.1 × 30 ÷ 15	**0.2**
Orchid, risk 40%, correlation 0.6	0.6 × 40 ÷ 15	**1.6**
From data: market 4, −2, 6, 0, 2; share 7, −3, 9, −1, 3	means 2 and 3; squares 40; cross-products 64	**covariance 64 ÷ 4 = 16; variance 40 ÷ 4 = 10**
Beta from the data	16 ÷ 10	**1.6**

Watch out: Treating volatility as beta (Quill and Tern swing equally but have betas 0.2 and 1.2); believing diversification removes market risk; treating a beta as a fact.

3. Portfolio beta

- A portfolio's beta is the weighted average of its holdings' betas. Betas combine this way because covariance with the market adds up; standard deviations do not, because of correlation.
- Portfolio beta is the exposure to the market. To lower it move into low beta shares or cash; to raise it move into high beta shares. It describes an average tendency, not a promise for any one period.

Portfolio beta = $w_1 \times \beta_1 + w_2 \times \beta_2 + w_3 \times \beta_3 + \ldots$

TERN 40% (B 1.2), LINDEN 40% (B 0.8), ORCHID 20% (B 1.6) IN $1M

Tern	0.4 × 1.2	**0.48**
Linden	0.4 × 0.8	**0.32**
Orchid	0.2 × 1.6	**0.32**
Portfolio beta	0.48 + 0.32 + 0.32	**1.12**
If the market falls 10%	1.12 × −10% on $1m	**−11.2% ($112,000)**

4. The CAPM and the security market line

- Required return is what an investor must be offered to hold an asset, and the rate used to discount its cash flows. The risk-free rate pays for waiting; the market risk premium (equity risk premium, expected market return minus the risk-free rate) is scaled by beta.
- Beta 1 earns the market's expected return; beta 0 earns only the risk-free rate. Nothing is paid for risk that diversifies away.
- The security market line plots required return against beta: intercept at the risk-free rate, slope equal to the market premium, every fairly priced asset on the line.
- Beta multiplies the premium, never the market return.

Required return = Rf + β × (E(Rm) − Rf)

RF = 3%, E(RM) = 8%, SO THE PREMIUM IS 5%		
Tern, beta 1.2	3% + 1.2 × 5%	**9.0%**
Linden, beta 0.8	3% + 0.8 × 5%	**7.0%**
Orchid, beta 1.6	3% + 1.6 × 5%	**11.0%**
Quill, beta 0.2	3% + 0.2 × 5%	**4.0%**
Rf 2%, market 8%, beta 0.5: right way	premium 6%; 0.5 × 6% = 3%; 2% + 3%	**5.0%**
The two slips	0.5 × 8%; 2% + 0.5 × 8%	**4.0%; 6.0%**

Watch out: Multiplying beta by the whole market return instead of the premium.

5. Alpha: cheap or dear

- Alpha is the analyst's forecast (expected return) minus the required return from the line. Above the line, alpha is positive and the asset looks undervalued; below it, negative and overvalued; on it, zero and fairly priced.
- Each forecast is compared with the required return at that share's own beta.
- A gap tends to close through price: at a price giving an expected return above the line, buying pressure pushes the price up to the level where the expected return falls onto the line. The fair price is the payoff divided by one plus the required return.
- Alpha is only as good as the forecast. A huge alpha usually means the forecast is too hopeful or the beta is mismeasured. Check costs and taxes and trade only if confident.

Alpha = Expected return − Required return

FORECASTS AGAINST THE LINE		
Tern: 11.0% − 9.0%	above the line	**+2.0%, undervalued**
Linden: 6.0% − 7.0%	below the line	**−1.0%, overvalued**
Orchid: 11.0% − 11.0%	on the line	**0.0%, fair**
Tern payoff $55.50 at price $50	55.50 ÷ 50 − 1; fair price 55.50 ÷ 1.09	**11.0%; $50.92**
Linden payoff $42.40 at price $40	42.40 ÷ 40 − 1; fair price 42.40 ÷ 1.07	**6.0%; $39.63**

6. Assumptions, Roll's critique and what came next

- Assumptions: investors are rational and risk averse and judge portfolios by expected return and variance only; they share expectations and a single period.
- Markets are frictionless (no taxes, costs or short-sale limits, divisible assets, price takers) and all can borrow and lend freely at one risk-free rate. Then everyone holds the market portfolio of every risky asset.
- Roll's critique: the true market portfolio (shares, bonds, property, private businesses, human capital) cannot be observed, so tests use a stand-in index, which brings benchmark error.
- If the stand-in is mean-variance efficient a straight line appears by definition; if not, none appears. Every test is a joint test of the model and the proxy.
- Size, value and momentum earned more than beta predicted, which motivated multifactor models with a premium for each source of risk. The CAPM's lasting value is the question: is this a risk the market pays me to carry?

A GAP THE MODEL CANNOT EXPLAIN (RF 3%, PREMIUM 5%, SMALL-COMPANY PORTFOLIO, BETA 1.0)

CAPM required return	3% + 1.0 × 5%	**8.0%**
Actually earned (example)	average over many years	**10.0%**
Gap the model misses	10.0% − 8.0%	**+2.0%**

Key takeaways

- Systematic risk cannot be diversified away; nonsystematic risk can, so only systematic risk is paid.
- Beta = ρ × σshare ÷ σmarket = covariance ÷ market variance; it is the slope against the market.
- Portfolio beta is the weighted average of the holdings' betas.
- CAPM: required return = Rf + β × (E(Rm) − Rf); beta multiplies the premium only.
- Alpha = expected − required return: positive above the security market line (cheap), negative below (dear).
- The CAPM rests on strong assumptions and cannot be tested cleanly (Roll's critique); hold its answers loosely.

Quick check

1. Share risk 25%, market risk 20%, correlation 0.8. Beta?
   Answer: 0.8 × 25 ÷ 20 = 1.0.
2. Rf 2%, market risk premium 4%, beta 1.5. Required return?
   Answer: 2% + 1.5 × 4% = 8.0%.
3. Rf 3%, market 8%, beta 1.2, forecast 8%. Alpha?
   Answer: Required 9%; alpha −1.0%; overvalued.