CHARTA LEARNING · LESSON NOTES

Performance measures and return-generating models

CFA Level I · Portfolio Management

You will learn: How to judge a portfolio's return against its risk with the Sharpe ratio, M squared, the Treynor ratio and Jensen's alpha, why they can disagree, and how the market model explains where a return comes from.

Running example: Examples use made-up round numbers. The risk-free rate is 2%. The market returned 10% with standard deviation 20% and beta 1. Portfolio A (Alder): return 14%, risk 30%, beta 0.8. Portfolio B (Birch): return 11%, risk 15%, beta 0.7.

1. Judging return against risk

- Same return does not mean same skill: a fair score judges return against the difficulty of the course, which is risk.
- Excess return is the portfolio return minus the risk-free rate (Rf), the reward for taking risk. Total risk is the standard deviation (σ), which counts every wobble. Beta counts only the market-driven, systematic part. The rest is specific risk, which can be diversified away.
- The capital asset pricing model (CAPM) says a fair expected return depends only on beta: Rf plus beta times the market risk premium (market return minus Rf). Beating the CAPM return is added value.

CAPM return = Rf + β × (Rm − Rf)

2. Sharpe ratio and M squared (total risk)

- Sharpe ratio is reward per unit of total risk (excess return ÷ standard deviation). Higher is better. It is the right ruler when the portfolio is the investor's whole pot. It ranks; it is not a percentage return. It counts specific risk too.
- M squared (M^2) puts every portfolio at the market's risk level: mix in borrowing if calmer than the market, hold cash if wilder, then compare plain returns. It is a return, directly comparable with the market's (some books subtract the market return to give a gain or shortfall).
- Because M^2 = Rf + Sharpe × σm and σm is the same for all portfolios, a higher Sharpe always means a higher M^2. The two can never disagree on ranking; they differ only in presentation.

Sharpe = (Rp − Rf) ÷ σp; M^2 = Rf + Sharpe × σm = Rf + (Rp − Rf) × σm ÷ σp

SHARPE, M^2, AND THE DIAL TURNED BY HAND

Sharpe: market / A / B	(10 − 2) ÷ 20; (14 − 2) ÷ 30; (11 − 2) ÷ 15	**0.40 / 0.40 / 0.60**
M^2: A	2 + 0.40 × 20	**10.0% (level with market)**
M^2: B	2 + 0.60 × 20	**14.0% (+4.0 points over market)**
B scaled up 20 ÷ 15 = 1.333	1.333 × 11 − 0.333 × 2	**14.0%**
A scaled down 20 ÷ 30 = 0.667	0.667 × 14 + 0.333 × 2	**10.0%**

3. Treynor ratio and Jensen's alpha (beta)

- Treynor ratio is reward per unit of systematic risk (excess return ÷ beta). It ignores specific risk on the view that the portfolio is one slice of a well-diversified whole where specific wobbles cancel. Higher is better.
- Jensen's alpha is not a ratio but a gap in percentage points: the actual return minus the CAPM return for the portfolio's beta. Positive means more than its market risk alone would have paid; zero is fair; negative is less. On the security market line, above the line is good.
- Slips: Sharpe and M² use standard deviation, Treynor uses beta, alpha uses beta inside the CAPM return. Use excess return (not plain return) on top for Sharpe and Treynor. Alpha subtracts the whole CAPM return, not just Rf. M² is a return; Sharpe and Treynor are ratios.

Treynor = (Rp − Rf) ÷ βp; Jensen's alpha = Rp − [Rf + βp × (Rm − Rf)]

MARKET RISK PREMIUM 10 − 2 = 8

Treynor: market / A / B	8 ÷ 1.00; 12 ÷ 0.80; 9 ÷ 0.70	**8.0 / 15.0 / 12.9**
A: CAPM return, alpha	2 + 0.8 × 8 = 8.4; 14 − 8.4	**alpha 5.6%**
B: CAPM return, alpha	2 + 0.7 × 8 = 7.6; 11 − 7.6	**alpha 3.4%**

4. Why the measures disagree, and which to use

- Scorecard: Sharpe and M² favour B (0.40 vs 0.60; 10.0% vs 14.0%). Treynor and alpha favour A (15.0 vs 12.9; 5.6% vs 3.4%). Nothing is wrong: they answer different questions.
- Split each portfolio's variance into a market part (β² × market variance) and a specific part. A is mostly specific risk, so total-risk measures charge it for all of it and it looks poor; beta-based measures charge only for beta, so it looks excellent. B is nearly all market risk.
- Whole holding, total risk matters: Sharpe or M². One sleeve in a diversified whole: Treynor or alpha. The exam clue is the wording: the whole portfolio, or one part of it.

MARKET VARIANCE = $20^2 = 400$

A: market part	$0.8^2 \times 400$	**256**
A: total variance, share from market	$30^2 = 900$; 256 ÷ 900	**28.4%**
B: market part	$0.7^2 \times 400$	**196**
B: share from market	196 ÷ 15^2 = 196 ÷ 225	**87.1%**

5. The market model

- A return-generating model writes a return as a recipe: a fixed bit, factor moves and a leftover surprise. The simplest has one factor, the market (the market model, or single-index model).
- Alpha is the intercept (return when the market return is zero); beta is the slope; beta × market return is the systematic part; the residual is the firm's own news, assumed to average zero (not zero each period) and unrelated to the market and to other firms' residuals.
- Alpha and beta come from a regression (best straight line through stock return against market return). Beta = correlation × σi ÷ σm = covariance ÷ market variance; they are the same number. The fitted model forecasts, and R squared measures how much the market explains.

Ri = αi + βi × Rm + ei

FIVE PERIODS: MARKET 2, 4, 6, 8, 10; STOCK 3, 5, 9, 10, 13 (%)

Averages		**market 6, stock 8**
Products of deviations; squared market deviations	20+6+0+4+20; 16+4+0+4+16	**50; 40**
Beta, alpha	50 ÷ 40; 8 − 1.25 × 6	**1.25; 0.5%**
Forecast if market is 8%	0.5 + 1.25 × 8	**10.5% (residual when stock made 10: −0.5)**
R squared	1 − 1.5 ÷ 64	**0.977**

6. Splitting risk, and multifactor models

- Variance splits into systematic risk (shared) and specific risk (the firm's own). Because residuals of different firms are unrelated, the covariance of two stocks needs only their betas and the market variance.
- With 100 stocks, direct estimation needs 4,950 covariances; the market model needs 100 betas, 100 residual variances and one market variance: 201 numbers.
- Multifactor models add factors. Macroeconomic factor models use surprises (not levels) in growth or inflation. Fundamental factor models use company characteristics such as size and value.
- The Fama and French model uses the market plus a size and a value factor. More factors do not guarantee a better model; they need more estimates.
- The market model assumes unrelated residuals; two banks hit by the same new rule would break that.

> **$\sigma i^2 = \beta i^2 \times \sigma m^2 + \sigma e^2$; $Cov(i, j) = \beta i \times \beta j \times \sigma m^2$**

ΣM = 20; STOCK I: CORRELATION 0.8, Σ 30; STOCK J: BETA 0.8, Σ 25

Beta of i	0.8 × 30 ÷ 20	**1.20**
i: market part, residual variance	$1.2^2 \times 400$; 900 − 576	**576; 324**
j: residual variance	$625 - 0.8^2 \times 400$	**369**
Covariance of i and j	1.2 × 0.8 × 400	**384**

Key takeaways

- Judge return against risk, using excess return over the risk-free rate.
- Sharpe (excess return ÷ σ) and M^2 (R_f + Sharpe × σ_m) use total risk and always rank alike.
- Treynor (excess return ÷ β) and Jensen's alpha (actual − CAPM return) use beta only.
- Whole portfolio: use Sharpe or M^2. One sleeve of a diversified whole: use Treynor or alpha.
- Market model: $R = \alpha + \beta \times R_m + e$; variance = $\beta^2 \times \sigma m^2$ + residual variance.
- Covariance between two stocks = $\beta i \times \beta j \times$ market variance, which cuts the number of estimates needed.

Quick check

1. R_p 9%, σ 12%, R_f 3%, σ_m 16%. Sharpe and M^2?
   Answer: 6 ÷ 12 = 0.5; M^2 = 3 + 0.5 × 16 = 11%
2. R_p 12%, β 1.5, R_f 2%, R_m 8%. Jensen's alpha?
   Answer: CAPM = 2 + 1.5 × 6 = 11%, so α = 1%
3. Correlation 0.6, σ_i 25%, σ_m 20%. Beta of the stock?
   Answer: 0.6 × 25 ÷ 20 = 0.75