CHARTA LEARNING · LESSON NOTES

Portfolio return, risk and correlation

CFA Level I · Portfolio Management

You will learn: How to find the expected return and risk of a two-asset portfolio, why a lower correlation means more diversification, what happens as holdings are added, and how the main asset classes have behaved over long periods.

Running example: Examples follow the made-up Linden Fund: a stock fund (expected return 10%, standard deviation 20%) and a bond fund (4%, 8%), correlation 0.2, with $600k in stocks and $400k in bonds.

1. Expected return: a weighted average

- A weight is the share of the portfolio's money held in one asset, written as a decimal. The weights must add up to one.
- Portfolio expected return is each asset's expected return times its weight, added up. Correlation plays no part in it, so return is a plain mix.
- The formula can be used backwards to hit a target return: write the stock weight as w and the bond weight as 1 − w, then solve for w.
- Traps: using a simple average when the weights are unequal, and weights that do not add to one.

$E(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2)$, with $w_1 + w_2 = 1$

LINDEN FUND RETURN, AND A 7% TARGET

Weights	600,000 ÷ 1,000,000; 400,000 ÷ 1,000,000	**0.6 and 0.4**
Expected return	0.6 × 10% + 0.4 × 4% = 6.0% + 1.6%	**7.6%**
Move to 30% stock	0.3 × 10% + 0.7 × 4%	**5.8%**
Target 7%	4% + w × 6% = 7%, so w × 6% = 3%	**w = 0.5**

2. Portfolio variance: three terms

- Variance is the average squared distance from the average; standard deviation is its square root. Covariance says whether two assets tend to swing together (positive) or apart (negative).
- Portfolio variance has three terms: each asset's variance times its weight squared, plus a teamwork term of 2 × the two weights × the covariance. Take the square root at the end to get standard deviation.
- Covariance equals correlation times the two standard deviations.
- Traps: leaving the weights unsquared (return uses the weights, variance uses the weights squared), dropping the 2, mixing percentages and decimals, forgetting the square root, and averaging standard deviations (right only when correlation is +1).

$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 \times w_1 \times w_2 \times Cov(1,2)$, with $Cov(1,2) = \rho \times \sigma_1 \times \sigma_2$

LINDEN FUND AT 60 : 40 (DECIMALS)

Stock term	$0.6^2 \times 0.04$	**0.014400**
Bond term	$0.4^2 \times 0.0064$	**0.001024**
Covariance	0.2 × 0.20 × 0.08	**0.0032**
Teamwork term	2 × 0.6 × 0.4 × 0.0032	**0.001536**
Variance	0.0144 + 0.001024 + 0.001536	**0.01696**
Standard deviation	√0.01696	**13.02%**
Benefit	weighted average 15.2% − 13.02%	**2.18 points**

3. Correlation and diversification

- Correlation is covariance rescaled by the two standard deviations, so it has no units and runs from −1 (always opposite) through 0 (no straight-line link) to +1 (lockstep). It is estimated from past returns using the sample formulas that divide by n − 1.
- A diversification benefit exists whenever correlation is below +1, and it grows as correlation falls. At +1, risk is exactly the weighted average. Zero is not the best case; −1 is.
- At −1 there is a mix with no risk: stock weight = bond standard deviation ÷ (sum of the two). Real assets never reach −1, so this is a limit, not a strategy.
- Judge an asset by how it moves with the rest of the portfolio, not by its own risk: adding a riskier asset can lower portfolio risk.
- Correlations are estimates and tend to rise in a crisis, when investors sell everything at once. Plan with a higher correlation than the one measured.

$\rho = \text{Cov}(1,2) \div (\sigma_1 \times \sigma_2)$, with $-1 \le \rho \le +1$

RISK OF THE 60 : 40 MIX AT DIFFERENT CORRELATIONS (RETURN STAYS 7.6%)

ρ = +1 / +0.5 / 0		**15.2% / 13.9% / 12.4%**
ρ = −0.5 / −1		**10.8% / 8.8%**
Zero-risk mix at ρ = −1	stock weight 8 ÷ (20 + 8)	**28.57% stock, return 5.71%**
Add 10% stock to bonds, ρ = 0	√(0.005184 + 0.0004) = √0.005584	**7.47% (below 8%), return 4.6%**
Crisis, ρ = 0.8	variance 0.0144 + 0.001024 + 0.006144 = 0.021568	**14.69% (was 13.02%)**

4. Many assets: the covariance floor

- For n assets the variance sum has n own-variance terms but n × (n − 1) covariance terms (each pair counted in both orders). At 100 assets it is 100 against 9,900, so covariances take over.
- With equal weights of 1/n, the own-variance piece fades toward zero as n grows, while the covariance piece settles at the average covariance. That level is a floor no extra holding removes.
- Idiosyncratic (specific) risk belongs to one company and is diversified away. Systematic (market) risk hits everything at once and stays; the average covariance measures it.
- In the long run an investor is paid only for risk that cannot be diversified away (the idea behind the capital asset pricing model). Five hundred holdings are not diversified if they share the same exposure.

$\sigma p^2 = (1 \div n) \times \text{average variance} + ((n - 1) \div n) \times \text{average covariance}$

EVERY VARIANCE 0.04, EVERY COVARIANCE 0.01 (MADE UP)

1 asset	0.04	**20.00%**
10 assets	0.004 + 0.9 × 0.01	**11.40%**
30 assets	0.0013 + 0.0097	**10.49%**
100 assets	0.0004 + 0.0099	**10.15%**
Endlessly many	0 + 0.01	**10.00%**

5. Risk and return in the long-run record

- In an illustrative record (made-up round numbers copying the long-run pattern), average return and risk both climb from Treasury bills to long-term bonds to large stocks to small stocks: 3%/3%, 5%/10%, 10%/18%, 14%/30%. More reward comes with more risk on average over long spans, not in every year.
- Nominal return is in money terms; real return is after inflation. Subtracting inflation is only an approximation, and the error grows as inflation rises.
- The arithmetic mean is the simple average of yearly returns. The geometric mean is the compound growth an investor actually lives on. It is always lower unless every year is the same, and the gap widens with volatility.
- Cautions: averages are not promises and one decade can reverse the order; stock returns show fat tails and negative skewness, which standard deviation understates; survivorship bias flatters a long record; quote real and geometric returns.

Real = (1 + nominal) ÷ (1 + inflation) − 1; Geometric ≈ arithmetic − σ² ÷ 2

REAL RETURNS AT 3% INFLATION, AND THE PRICE OF VOLATILITY

Bills / long bonds	1.03 ÷ 1.03 − 1; 1.05 ÷ 1.03 − 1	**0.00% / 1.94%**
Large / small stocks	1.10 ÷ 1.03 − 1; 1.14 ÷ 1.03 − 1	**6.80% / 10.68%**
$100 after +30%, then −20%	100 × 1.30 = 130; 130 × 0.80	**$104, not $110**
Arithmetic mean	(30% − 20%) ÷ 2	**5.00%**
Geometric mean	√1.04 − 1	**1.98%**

Key takeaways

- Expected return is a weighted average of the parts and does not depend on correlation.
- Variance squares the weights and adds 2 × w_1 × w_2 × covariance; take the square root last.
- Risk falls below the weighted average whenever correlation is below +1, and falls further as correlation falls.
- Judge a new asset by its covariance with what you hold, not its risk alone.
- As holdings multiply, own risk fades but the average covariance (systematic risk) remains as a floor.
- Higher risk means higher return on average, never a guaranteed return; use real and geometric returns.

Quick check

1. Weights 70% and 30%, returns 12% and 6%. Expected return of the portfolio?
   Answer: 0.7 × 12% + 0.3 × 6% = 10.2%.
2. Equal weights, both risks 20%, correlation 0. Portfolio standard deviation?
   Answer: Variance 0.02, so risk = 14.14%.
3. Four equal holdings, average variance 0.04, average covariance 0.01. Variance?
   Answer: 0.04 ÷ 4 + 0.75 × 0.01 = 0.0175.