CHARTA LEARNING · LESSON NOTES

# Is it worth it? NPV, IRR and returns

CFA Level I · Quantitative Methods

**You will learn:** Judge a project by its net present value and internal rate of return, measure holding period, time-weighted and money-weighted returns, read the yield and growth that prices imply, and find a forward rate.

**Running example:** Examples use made-up figures in pounds, such as a project that costs £500 today and pays £200 at the end of each of the next three years when you require 8% a year.

## 1. Net present value (NPV)

- Money later is worth less than money now, by an amount that depends on the rate you could earn elsewhere. Discounting shrinks a future payment back to today; the result is its present value. The rate is the required return: the minimum yearly return you would accept for the risk.
- Discount every future cash flow, add them up, and subtract the cost. The cash flow at time zero is the cost, written negative. Further away means more shrinking.
- Decision rule: positive NPV, accept; negative, reject. Do not just add cash flows from different dates.

$$NPV = CF_0 + CF_1 \div (1 + r)^1 + CF_2 \div (1 + r)^2 + \ldots$$

COST £500; £200 A YEAR FOR 3 YEARS; REQUIRED RETURN 8%

| | | |
|---|---|---|
| **Year 1** | 200 ÷ 1.08 | **185.19** |
| **Year 2** | 200 ÷ 1.08² | **171.47** |
| **Year 3** | 200 ÷ 1.08³ | **158.77** |
| **Present value of inflows** | sum, unrounded | **515.42** |
| **NPV** | 515.42 − 500 | **+15.42, accept** |

**Watch out:** Adding payments suggested a £100 profit; after allowing for waiting only about £15 is real.

## 2. Internal rate of return (IRR)

- The IRR is the discount rate that makes the NPV exactly zero: the project's own built-in yearly return. If the IRR is above the required return, accept; if below, reject. There is no tidy formula: use trial and error or a financial calculator.
- When you must choose one of two projects and NPV and IRR rank them differently, pick the higher NPV. NPV is measured in money, and the IRR quietly assumes cash received along the way can be reinvested at the IRR itself, which may be unrealistic.

SAME PROJECT: TRY RATES UNTIL NPV HITS ZERO

| | | |
|---|---|---|
| **NPV at 8%** | | **+15.42** |
| **NPV at 9%** | | **+6.26** |
| **NPV at 10%** | | **−2.63** |
| **IRR** | between 9% and 10% | **about 9.70%** |
| **IRR vs required 8%** | 9.70 > 8 | **Accept** |

**Watch out:** Choosing the higher IRR over the higher NPV.

## 3. Holding period return (HPR)

- The HPR is the total return over whatever time you held an investment. It counts the change in price and any income received, such as a dividend. Leaving out income understates the return. It says nothing about how long the period was.
- To join several periods, multiply growth factors; do not add the returns. The steady yearly rate is the nth root of the total growth factor, minus 1.

> **HPR = $(P_1 - P_0 + D_1) \div P_0$**

| SHARE BOUGHT AT £50, SOLD AT £54, DIVIDEND £2; AND CHAINING +12%, −5%, +8% | | |
|---|---|---|
| **Total gain** | (54 − 50) + 2 | **£6** |
| **HPR** | 6 ÷ 50 | **12%** |
| **Price part; dividend part** | 4 ÷ 50; 2 ÷ 50 | **8%; 4%** |
| **3-year return** | 1.12 × 0.95 × 1.08 − 1 | **14.91% (adding gives 15%, wrong)** |
| **Steady yearly rate** | 1.14912^(1/3) − 1 | **4.74%** |

**Watch out:** Adding yearly returns, or leaving out income.

## 4. Time-weighted and money-weighted returns

- How good was the manager, and how did the investor do? These have different answers because the client, not the manager, chooses when to deposit.
- Time-weighted return cuts the period at every deposit or withdrawal, takes the HPR of each piece, adds 1 to each, multiplies them and subtracts 1. Deposits do not matter, so it judges the manager.
- Money-weighted return is the IRR of the client's own cash flows (money in, money out, final value). Deposits and timing matter a great deal, so it describes the investor's experience.

> **Time-weighted = $(1 + r_1) \times (1 + r_2) \times \ldots - 1$. Money-weighted = IRR of the client's cash flows**

| BUY A SHARE AT £100; YEAR 1 PRICE £150, BUY ANOTHER; YEAR 2 PRICE £120 | | |
|---|---|---|
| **Year 1 return** | 150 ÷ 100 − 1 | **+50%** |
| **Year 2 return** | 120 ÷ 150 − 1 | **−20%** |
| **Time-weighted return** | 1.50 × 0.80 − 1 | **20%** |
| **Client's cash flows** | −100, −150, +240 | |
| **Money-weighted return** | rate where NPV = 0 | **−2.88%** |
| **Gap** | 20 − (−2.88) | **22.88 points** |

**Watch out:** Using the money-weighted return to judge a manager, or the time-weighted return to describe what the client earned.

## 5. Solving pricing formulas backwards

- Given a price, ask what return it promises. For a single-payment bond, yield = (face ÷ price)^(1/n) − 1. For a stock, the simplest model gives implied growth g = r − $D_1 \div P_0$, where $D_1 \div P_0$ is the dividend yield.
- A bond with several payments has no neat rearrangement: try yields until the price matches. A bond priced above par has a yield below its coupon rate.
- $D_1$ must be next year's dividend. If you are given the dividend just paid, $D_1 = D_0 \times (1 + g)$, which puts g on both sides.

**Bond: Price = Face ÷ $(1 + y)^n$. Stock: $P_0 = D_1 \div (r - g)$**

IMPLIED YIELD AND GROWTH

| | | |
|---|---|---|
| **Zero-coupon bond, face £1,000, 5 years, price £747.26: growth** | 1,000 ÷ 747.26 | **1.3382** |
| **Implied yield** | 1.3382^(1/5) − 1 | **6.00%** |
| **3-year bond, 6% coupon, face 100, price 102.72: try 6% / 4% / 5%** | price comes to | **100.00 / 105.55 / 102.72** |
| **Implied yield** | price matches | **5.00%** |
| **Stock £50, $D_1$ = £2, required 9%: implied g** | 9.00% − 2 ÷ 50 | **5.00%** |
| **If £2 was just paid ($D_0$)** | (4.50 − 2.00) ÷ 52 | **g = 4.81%** |

**Watch out:** Using the dividend just paid instead of next year's when finding implied growth.

## 6. Cash flow additivity and forward rates

- Cash flow additivity: the value of a set of cash flows is the sum of the value of each one. If not, you could buy the package, sell the pieces and keep a riskless profit (arbitrage), which markets do not leave lying around.
- A spot rate is the rate for money lent from today for a given number of years. A forward rate is the rate for a future period, fixed today. Compare two routes to the same end point; they must give the same amount.
- A coupon bond is priced as separate zero-coupon payments, each discounted at its own spot rate. A forward rate is not the second spot rate plus a gap: divide the growth factors.

**$(1 + z_2)^2 = (1 + z_1) \times (1 + f)$, so $f = (1 + z_2)^2 \div (1 + z_1) - 1$**

ONE-YEAR SPOT 4%, TWO-YEAR SPOT 5%

| | | |
|---|---|---|
| **Lock in two years** | $1.05^2$ | **1.1025** |
| **Divide by 1.04** | 1.1025 ÷ 1.04 | **1.0601** |
| **Forward rate f** | 1.0601 − 1 | **6.01%** |
| **Two-year bond, 6% coupon: year 1** | 6 ÷ 1.04 | **5.77** |
| **Year 2** | $106 \div 1.05^2$ | **96.15** |
| **Bond price** | sum, unrounded | **101.91** |

## Key takeaways

- NPV discounts every cash flow, adds them, subtracts the cost: positive means accept.
- IRR is the rate that makes NPV zero; if NPV and IRR disagree, follow NPV.
- HPR = $(P_1 - P_0 + D_1) \div P_0$, and years are chained by multiplying, not adding.
- Time-weighted return judges the manager; money-weighted return (the client's IRR) describes the investor.
- Implied yields and growth rates come from solving the pricing formulas backwards, using next year's dividend.
- A forward rate comes from setting two routes equal, because additivity rules out free money.

## Quick check

1. Project A has the higher IRR, project B the higher NPV. You can do only one. Which?
   **Answer:** B. Follow NPV: it is in money and its reinvestment assumption is more realistic.
2. Buy a share at £40, receive £1 dividend, sell at £42. Holding period return?
   **Answer:** (42 − 40 + 1) ÷ 40 = 7.5%.
3. One-year spot 3%, two-year spot 4%. Implied one-year forward rate, one year ahead?
   **Answer:** $1.04^2 \div 1.03 - 1 = 5.01\%$.

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Summary notes for the Charta whiteboard lesson. Examples use made-up figures. Watch the full lesson at charta-learning.com.