CHARTA LEARNING · LESSON NOTES

Return and risk of a portfolio

CFA Level I · Quantitative Methods

You will learn: Work out the expected return, variance and standard deviation of a two-asset portfolio, explain why lower correlation means more diversification, and find the minimum-variance weights.

Running example: Examples use made-up figures. Asset A (or 1) has an expected return of 8% and a standard deviation of 20%; asset B (or 2) has 12% and 30%; the correlation is 0.25; the first mix holds 60% in A and 40% in B.

1. Portfolio return

- Expected return is the average result you would expect over many tries. A weight is the share of your money in each asset, and the weights add up to 100%: $w_1 + w_2 = 1$.
- Portfolio return is a plain weighted average, so it lands between the two asset returns, closer to the one you hold more of.

$E(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2)$

60% IN A, 40% IN B

Asset A share	0.6 × 8%	**4.8%**
Asset B share	0.4 × 12%	**4.8%**
Portfolio return	4.8 + 4.8	**9.6%**

2. Risk, covariance and correlation

- Risk is how much the return bounces around. Variance is the average squared distance from the average (squaring stops ups and downs cancelling). The standard deviation σ is its square root, in the same units as returns.
- Risk is not a weighted average: the mix is usually less risky than that, because wobbles partly cancel when assets move differently.
- Covariance shows whether two assets move in the same direction (positive), opposite directions (negative) or no pattern (zero), but its size depends on units. Correlation ρ rescales it to between −1 and +1: +1 copycats, 0 strangers, −1 seesaw. Lower correlation means more cancelling.

$\rho = Cov \div (\sigma_1 \times \sigma_2)$, so $Cov = \rho \times \sigma_1 \times \sigma_2$

COVARIANCE OF THE TWO ASSETS

Cov	0.25 × 0.20 × 0.30	**0.015**

3. Portfolio variance and standard deviation

- Three terms: asset 1 on its own (weight squared × own variance), asset 2 on its own, and the teamwork term holding the correlation. It is the only term that can be negative, and it is where diversification lives.
- Weights are squared, so spreading out shrinks the own terms fast. Take the square root at the end to get the standard deviation.
- The gap between the weighted average of the risks and the actual portfolio risk is the diversification benefit. It is free, and the return is unchanged.

$Var(R_p) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\, w_1\, w_2\, \rho\, \sigma_1\, \sigma_2$

60% IN A, 40% IN B, CORRELATION 0.25		
A on its own	0.36 × 0.04	**0.0144**
B on its own	0.16 × 0.09	**0.0144**
Teamwork term	2 × 0.6 × 0.4 × 0.25 × 0.06	**0.0072**
Variance, then risk	0.036, square root	**18.97%**
Weighted average of risks	0.6 × 20% + 0.4 × 30%	**24.00%**
Diversification benefit	24.00 − 18.97	**5.03 points**

Watch out: Averaging the risks (only true at ρ = +1), forgetting the square on the weights, and forgetting the square root at the end.

4. What correlation does

- At ρ = +1 there is no benefit: portfolio risk is the weighted average. At any ρ below 1 the mix is less risky than the weighted average, and the lower the correlation, the bigger the gain.
- At ρ = −1 the wobbles can cancel completely and risk can fall to zero. Diversification does not change the portfolio return.

50/50 mix of the two assets: Var = 0.0325 + 0.03ρ

50/50 MIX, RISKS 20% AND 30%		
Own terms	0.25 × 0.04 + 0.25 × 0.09	**0.0325**
Teamwork term	2 × 0.5 × 0.5 × ρ × 0.06	**0.03ρ**
ρ = +1.00	√0.0625	**25.00%**
ρ = +0.25	√0.04	**20.00%**
ρ = 0.00	√0.0325	**18.03%**
ρ = −1.00	√0.0025	**5.00%**

5. The minimum-variance portfolio

- The minimum-variance portfolio is the blend with the lowest variance. Risk traces a bowl as money slides from one asset to the other, and we want the bottom. A bit of the riskier asset can make the mix safer.
- Use variances, not standard deviations, in the formula. Square the risks first. Asset 2 gets the rest: $w_2 = 1 - w_1$.
- Checks: at ρ = 0 the weights are inverse to the variances. At ρ = −1, $w_1 = \sigma_2 \div (\sigma_1 + \sigma_2)$ and risk is zero. At ρ = +1 there is no bowl, so with no borrowing or short selling you hold only the less risky asset.
- Plotting return against risk, the minimum-variance portfolio is the tip of the curve. Above it is the efficient frontier; below it is the same risk for less return.

$w_1 = (\sigma_2^2 - Cov) \div (\sigma_1^2 + \sigma_2^2 - 2\, Cov)$

RISKS 20% AND 30%, P = 0.25		
Top, bottom	0.09 − 0.015; 0.04 + 0.09 − 0.03	**0.075; 0.100**
Weights	$w_1 = 0.075 \div 0.1$, $w_2 = 1 - w_1$	**75% / 25%**
Variance	0.0225 + 0.005625 + 0.005625	**0.03375**
Risk, return	√0.03375; 0.75 × 8% + 0.25 × 12%	**18.37%; 9.0%**

Watch out: Using σ instead of σ² in the weight formula, and assuming the safer asset alone is always the lowest-risk choice.

6. The variance trap and perfect opposites

- With ρ = 0 the covariance is zero and $w_1 = \sigma_2^2 \div (\sigma_1^2 + \sigma_2^2)$. Using the standard deviations by mistake gives 60% and 40%, which is riskier than the right answer. Both beat holding asset 1 alone at 20%, but only the lowest earns credit.
- With ρ = −1 the two wobbles are the same size and exactly opposite, so they cancel: a risk-free portfolio built from two risky assets. It is rare in real markets.

RISKS 20% AND 30%

ρ = 0, right weights	0.09 ÷ 0.13	**69.2% / 30.8%**
Right: variance, risk	own terms only	**0.0277; 16.64%**
Wrong weights (use risks)	0.30 ÷ 0.50	**60% / 40%**
Wrong: variance, risk	0.0144 + 0.0144	**0.0288; 16.97%**
ρ = −1, weights	w_1 = 0.30 ÷ (0.20 + 0.30)	**60% / 40%**
Wobbles, risk	0.6 × 20% = 12%; 0.4 × 30% = 12%	**12% − 12% = 0%**

Watch out: Mixing up covariance and correlation.

Key takeaways

- Portfolio return is the weighted average of the asset returns.
- Portfolio risk is not a weighted average; the teamwork term holds the correlation.
- Lower correlation means more diversification; at ρ = +1 there is none.
- $\text{Cov} = \rho \times \sigma_1 \times \sigma_2$, and variance (not standard deviation) goes into the minimum-variance formula.
- The minimum-variance weight is $w_1 = (\sigma_2^2 - \text{Cov}) \div (\sigma_1^2 + \sigma_2^2 - 2\,\text{Cov})$, and $w_2 = 1 - w_1$.
- At ρ = 0 the weights are inverse to the variances.

Quick check

1. 70% in an asset returning 10%, 30% in one returning 5%. Portfolio return?
   Answer: 7.0% + 1.5% = 8.5%.
2. Cov = 0.015, risks 20% and 30%. What is the correlation?
   Answer: 0.015 ÷ 0.06 = 0.25.
3. Equal risks, correlation 0. Minimum-variance weights?
   Answer: 50% each: equal variances, equal weights.