CHARTA LEARNING · LESSON NOTES

Types of financial returns and benchmarking returns

CFA Level I · Quantitative Methods

You will learn: How to annualise a return, strip a headline return down to net, after-tax and real figures, understand leverage, and compute the return of a price-weighted, market-cap-weighted or equal-weighted index, with and without income.

Running example: Examples use made-up figures. The Willow Fund earns 2% in a quarter, then 9% gross in a year with fees of 0.85%; the Meadow Index holds three shares: Aster ($100, 1m shares), Briar ($50, 10m shares) and Clover ($10, 20m shares), total market value $800m.

1. Annualising a return

- A return is the gain on an investment as a percentage of the money put in. Returns over different lengths of time cannot be compared directly. Annualising restates a return as the equivalent return for one year.
- r is the return earned in one period (as a decimal). m is how many such periods fit in a year: 4 for a quarter, 12 for a month, 365 ÷ days for a stretch of days. If the period is longer than a year, m is below 1.
- Compound, do not multiply. The multiplying shortcut is too low for periods under a year (each period's gain earns gains of its own) and too high for periods over a year.
- Annualising is a unit change, not a forecast. 3% in one week gives $1.03^{52} - 1 = 365\%$ a year, which assumes 51 more identical weeks. The shorter the period, the bigger and less reliable the headline, so ask how long it was measured.

Annualised return = $(1 + r)^m - 1$

WILLOW FUND

2% in a quarter (m = 4)	$1.02^4 - 1$	**8.24% (shortcut 8.00%)**
0.90% over 40 days, m = 365 ÷ 40 = 9.125	1.009^9.125 − 1 = 1.0852 − 1	**8.52% (shortcut 8.21%)**
£100 grows to £130 in 3 years, m = 1 ÷ 3	1.30^(1/3) − 1 = 1.0914 − 1	**9.14% (30% ÷ 3 = 10%, too high)**
0.5% a quarter / a month / a week	annualised	**2.02% / 6.17% / 29.61%**

Watch out: Multiplying instead of compounding, and treating an annualised week or month as a forecast.

2. Gross, net, after-tax and real returns

- Gross is the headline before anything is taken off. Net is after the manager's fees and costs. After-tax is net after the investor's own tax, so it depends on the investor, not the fund. Real is after inflation, the general rise in prices; nominal is the money return.
- Net return is approximately gross minus fees. After-tax is net times (1 − tax rate), the share the investor keeps, with tax charged on the whole return.
- The exact real return divides the growth factors, because rising prices reduce the worth of the whole ending amount. The shortcut, nominal minus inflation, is close at low inflation but a little too generous, and the gap grows when inflation is high.
- Never compare a gross figure for one fund with a net figure for another, or pre-tax with after-tax.

> **Net ≈ gross − fees. After-tax = net × (1 − tax rate). Real = (1 + nominal) ÷ (1 + inflation) − 1**

WILLOW FUND: 9.00% GROSS, FEES 0.85%, TAX 20%, INFLATION 3.1%

Net return	9.00 − 0.85	**8.15%**
Tax at 20%	8.15 × 0.20	**1.63%**
After-tax return	8.15 − 1.63	**6.52%**
Real return	1.0652 ÷ 1.031 − 1 = 1.0332 − 1	**3.32% (shortcut 3.42%)**
Pension fund, tax 0%	after tax 8.15%	**real 4.90%**
Taxpayer at 35%	after tax 5.30%	**real 2.13%**

Watch out: Subtracting inflation when the exact real return is asked: divide the growth factors, then subtract one.

3. Leverage

- Leverage means investing with borrowed money as well as your own. It does not change the underlying return; it magnifies what you make or lose on your own money, in both directions, and the interest is owed either way.
- R is the underlying return and the ratio is the amount borrowed ÷ your own money. Read the bracket as a test: if the investment beats the borrowing rate, leverage adds to your return; if not, it makes things worse.

> **Leveraged return = R + (borrowed ÷ own money) × (R − borrowing rate)**

OWN £100, BORROW £100 AT 4%, RATIO = 1

Underlying +9%	9 + 1 × (9 − 4)	**+14%**
Underlying 0%	0 + 1 × (0 − 4)	**−4%**
Underlying −9%	−9 + 1 × (−9 − 4)	**−22%**
Swing, unlevered	9 − (−9)	**18 points**
Swing, levered	14 − (−22)	**36 points**

Watch out: Forgetting that leverage cuts both ways: more return, and more risk.

4. Price-weighted index

- An index is a basket of investments boiled down to one number. The weighting rule decides how much each share moves it, so the same shares can give three different answers.
- A price-weighted index adds up the share prices and divides by a divisor (at the start usually the number of shares, so it is a simple average of prices). A share's weight is its price ÷ the sum of prices; company size never enters. The highest price steers.
- After a stock split the price drops but nothing real has changed, so the divisor is reset to keep the index level unchanged. The split also reduces that share's weight.

> **Index = sum of prices ÷ divisor. New divisor = new sum of prices ÷ old index value**

MEADOW INDEX: ASTER 100→110, BRIAR 50→45, CLOVER 10→12; DIVISOR 3

Index, start	(100 + 50 + 10) ÷ 3 = 160 ÷ 3	**53.33**
Index, end	(110 + 45 + 12) ÷ 3 = 167 ÷ 3	**55.67**
Price return	55.67 ÷ 53.33 − 1	**4.38%**
Aster splits 2 for 1: sum of prices	50 + 50 + 10	**110**
Index with old divisor 3	110 ÷ 3	**36.67 (a false fall)**
New divisor, then index	110 ÷ 53.33 = 2.0625; 110 ÷ 2.0625	**53.33**
Aster's weight	falls from 62.5% to	**45.5%**

5. Market-cap and equal-weighted indexes

- Market-capitalisation weighting counts each company by its size, as if you owned the whole market. Market value = price × number of shares; weight = a company's value ÷ total value. The biggest company steers. No divisor or rebalancing is needed.
- Equal weighting gives every share the weight 1 ÷ n, and the index return is the average of the share returns. It tilts the index towards smaller companies.
- Equal-weighted weights drift as prices move (the winner grows above 1 ÷ n), so the index must be rebalanced regularly: sell some winners, buy more losers.
- Same market, same year, three answers. Always ask how an index is weighted.

> **Cap-weighted return = ending total value ÷ starting total value − 1. Equal-weighted return = $(R_1 + R_2 + \ldots + R_n) \div n$**

MEADOW INDEX, SAME YEAR

Market values, start	100 + 500 + 200	**$800m**
Market values, end	110 + 450 + 240	**$800m**
Cap-weighted return	800 ÷ 800 − 1	**0.00%**
Share returns	Aster +10%, Briar −10%, Clover +20%	
Equal-weighted return	(10 − 10 + 20) ÷ 3	**6.67%**
Drift after one year	110, 90, 120	**34.4%, 28.1%, 37.5%**
Aster's weight	price / equal / cap weighting	**62.5% / 33.3% / 12.5%**

6. Price return and total return

- Price return uses only price changes. Total return also adds income, such as dividends (cash payments from a company to its shareholders).
- Divide by the beginning value to turn the gain into a percentage. With no income, total return equals price return; otherwise total return is higher and is the fairer yardstick for an investor.

> **Total return = (ending value − beginning value + income) ÷ beginning value**

BRIAR PAYS A $1 DIVIDEND PER SHARE (10M SHARES = $10M)

Cap-weighted, price return	800 ÷ 800 − 1	**0.00%**
Cap-weighted, total return	(800 − 800 + 10) ÷ 800	**1.25%**
Price-weighted, total return	(167 − 160 + 1) ÷ 160	**5.00%**
Equal-weighted, total return	(10 − 8 + 20) ÷ 3	**7.33%**

Key takeaways

- Annualised return = $(1 + r)^m - 1$: compound, never multiply, and it is a unit change, not a forecast.
- Gross to net removes fees, net to after-tax removes the investor's tax, nominal to real removes inflation.
- Exact real return = (1 + nominal) ÷ (1 + inflation) − 1.
- Leverage magnifies return and risk in both directions.
- Price-weighted: highest price steers and the divisor is reset after a split. Cap-weighted: biggest company steers. Equal-weighted: average of returns, needs rebalancing.
- Total return adds income to the price change.

Quick check

1. A fund earns 1.5% in a quarter. What is its annualised return?
   Answer: $1.015^4 - 1 = 6.14\%$.
2. Gross 7%, fees 1%, inflation 2%, ignore tax. Exact real return?
   Answer: Net 6%; 1.06 ÷ 1.02 − 1 = 3.92%.
3. Which index weights a share by price alone? What is reset after a split?
   Answer: Price-weighted; the divisor.